Modelling, Uncertainty and Data for Engineers (MUDE)

Optimization week. Plenary Lecture

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TU Delft

Optimization week – The team

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Outline

First part

1) **Optimization origins**
2) **Understanding the general components of an optimization problem**
3) **Optimization vs Simulation in modeling a real-world problem**
4) **Quiz**

Second part

1) **Examples of optimization problems**
2) **Types of optimization problems**

Where is Optimization coming from?

- In the UK and in the US, scientists started to be called between the first and second world Wars to collaborate with the military in doing **research** on military **operations**.

- A second world war was on the horizon and both countries wanted to be prepared by optimizing their logistics to maximize their chances of winning battles.

- They created a field of applied sciences known as **Operations Research** in which **Optimization** can be placed.

The radar

- Modern operations research originated in the UK in 1937 and was the result of an initiative of the superintendent, A. P. Rowe.

- Rowe conceived the idea to analyze and improve the working of the UK's early warning radar system, Chain Home (CH). Initially, he analyzed the operation of the radar equipment and its communication networks to provide a complete vision of the south coast of the UK. **How many radars do you need? Where should they be located?**

- The analysis was later expanded to include the operating personnel's behavior to plan the Human Resources (HR) of this system.

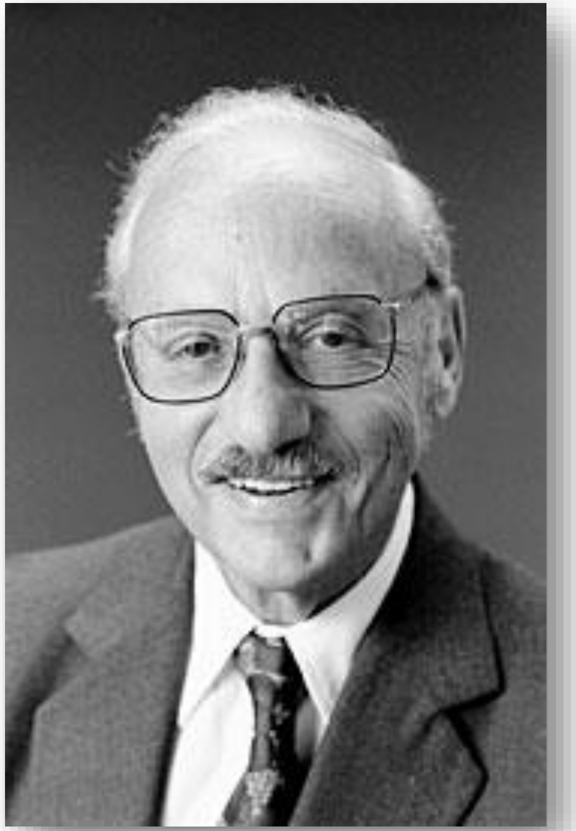
- <u>This revealed unappreciated limitations of the CH network and allowed remedial action to be taken which helped win the war.</u>

Chain Home transmitter antenna

1947: The Simplex method is developed

- George Dantzig worked on planning methods for the US Army Air Force during World War II using a desk calculator to find solutions to hard operational problems. **In 1946 he was challenged to mechanize/automate the planning process that he was using.**

- Dantzig formulated the planning problem, typically a problem of assigning resources to activities, as linear inequalities (or equalities) inspired by the work of Wassily Leontief, however, **<u>at that time he didn't include an objective as part of his formulation, he was mainly searching for feasible solutions to a problem.</u>**



George Dantzig
(1914-2005)

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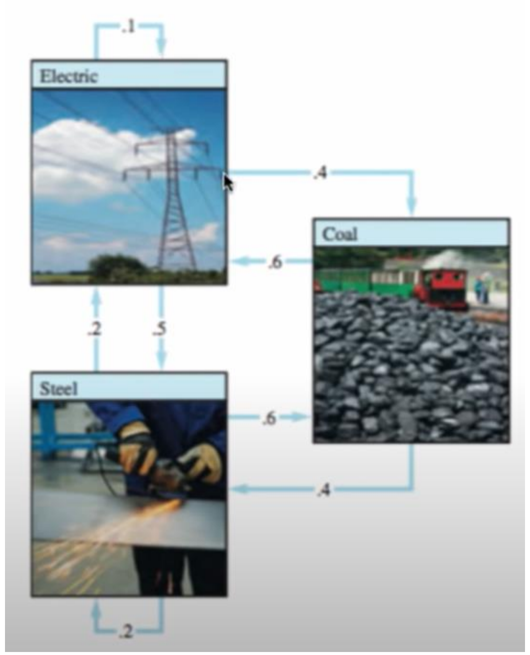
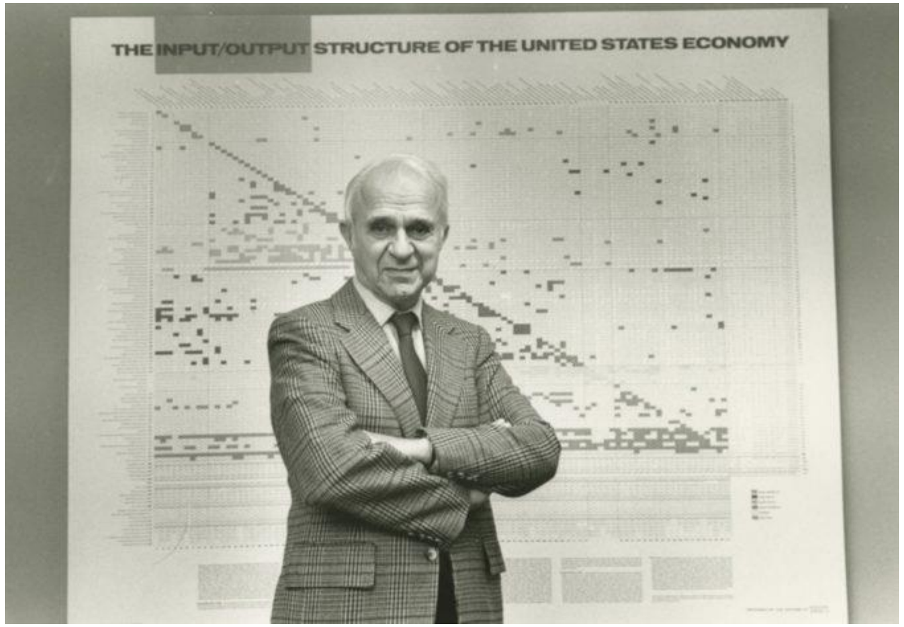
The work of Leontief

Input/Output matrix in a simple economy:

	Coal	Electricity	Steel
Coal	0	0.4	0.6
Electricity	0.6	0.1	0.2
Steel	0.4	0.5	0.2
	1	1	1

$$p_C = 0 \times p_C + 0.4 \times p_E + 0.6 \times p_S$$
$$p_E = 0.6 \times p_C + 0.1 \times p_E + 0.2 \times p_S$$
$$p_S = 0.4 \times p_C + 0.5 \times p_E + 0.2 \times p_S$$





Wassily Leontief
(1906-1999)

- In this example a simple economy is described through three main products/activities, Coal, Electricity, and Steel. It's easy to produce linear equations where the interdependencies between these products are apparent once you have the production factors (how much you need from each one of those to produce the other).
- Using the right data and this logic it's possible to describe the functioning of many systems and their corresponding "products".

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Adding an objective

- Without an objective, in many planning problems a vast number of solutions can be feasible, and therefore to find the "best" feasible solution, military-specified objectives **- don’t forget that Dantzig was studying military operations -** must be used that describe how goals can be achieved as opposed to specifying a specific value for this goal on itself. For example, it’s not about transporting 1000 soldiers but finding a way to transport as many as possible with the existing resources.

- Dantzig's core insight was to realize that most such ground objectives can be translated into a **linear objective function** that needs to be maximized (or minimized) which measures the quality/performance of the solutions.

- Development of the final method, the so-called **simplex** method, was evolutionary and happened over a period of about a year.

A linear program that can be solved through the simplex method

$$\max(F) = 2x + y$$

Objective function

$$\left.\begin{aligned} 3x + y &\leq 150 \\ x + y &\leq 90 \\ x &\geq 40 \\ y &= 20 \end{aligned}\right\}$$

Linear inequalities/equalities

$$x, y \geq 0$$

Variables domain

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x and y are continuous variables whose values we want to determine and in this case, they must be positive.

In general, we have, in a linear mathematical program

Objective function

Objective function coefficients

$$\text{Max (Min) } Z = c_1\, x_1 + c_2\, x_2 + \ \ldots + c_n\, x_n$$

subject to,

Decision variable

Constraints

$$a_{11}x_1 + a_{12}x_2 + \ \ldots + a_{1n}x_n \ (\leq, =, \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ \ldots + a_{2n}x_n \ (\leq, =, \geq)\ b_2$$
$$\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ \ldots + a_{mn}x_n \ (\leq, =, \geq)\ b_m$$

Technology coefficients, they measure the usage of a resource

Independent term

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n is used for the number of variables, and
m for the number of constraints

Optimization

- Some modeling approaches attempt to provide optimal answers for problems (e.g., mathematical programming) or near optimum answers (e.g., heuristic and meta-heuristic methods).

Simulation

- A simulation model predicts the performance of a system under a specific set of inputs (experimental parameters).In general, with simulation, we are not searching for an optimal solution but for the system’s performance under different scenarios that are selected according to their importance or likelihood.

A small problem?

- Imagine the case of planning a bus line through simulation. You have your route defined (the streets where it's going to go through) and the demand around that route is dependent on the frequency of the buses and the bus stop distance. You want to simulate the Bus operation in order to maximize your profit.

- Variables: Bus fleet size (b); Number of stops (s) in the line.

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- If we define the fleet dimension as a parameter that varies between 5 and 15 buses, we have 11 bus fleet dimensions to test in a simulation model.
- If we define the number of stops to be between 10 and 30, we have 21 possible stops' dimensions.
- We have to test b x s=231 combinations, we also call this the **total enumeration of the solutions** (231 is manageable in simulation)

- Imagine now that the route line is not designed yet (its shape -> the order of stops to visit) then for each combination of fleet and number of stops you would have to test the factorial of the number of bus stops which are all the combinations of stops that form a path ($(n-1)!/2$). For 30 stops this would be $(30-1)!/2 = 4.42 \times 10^{30}$.

This leads to an impractical number of scenarios to test in simulation!

This problem will be better studied with optimization techniques in the so-called network design problems which can be solved, to a certain extent, with mathematical programming as well or witj specific heuristics.

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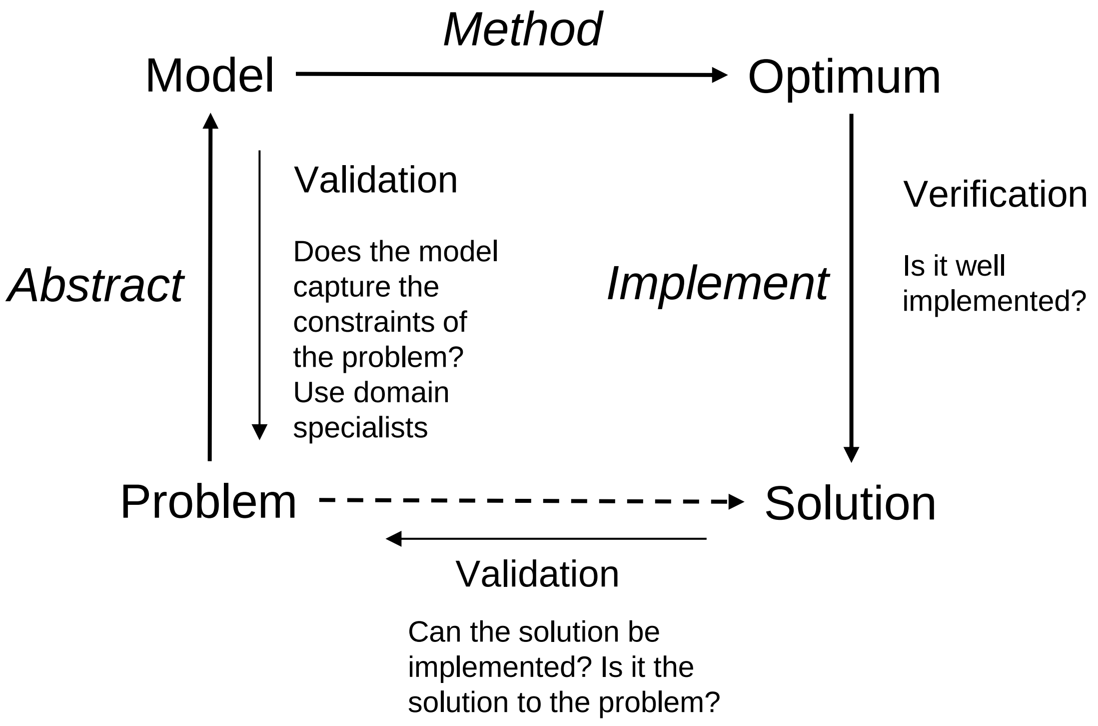
Computation power

5 MB computer being loaded on an airplane

500 GB computers “loaded” on an airplane

Typical workflow for an optimization study



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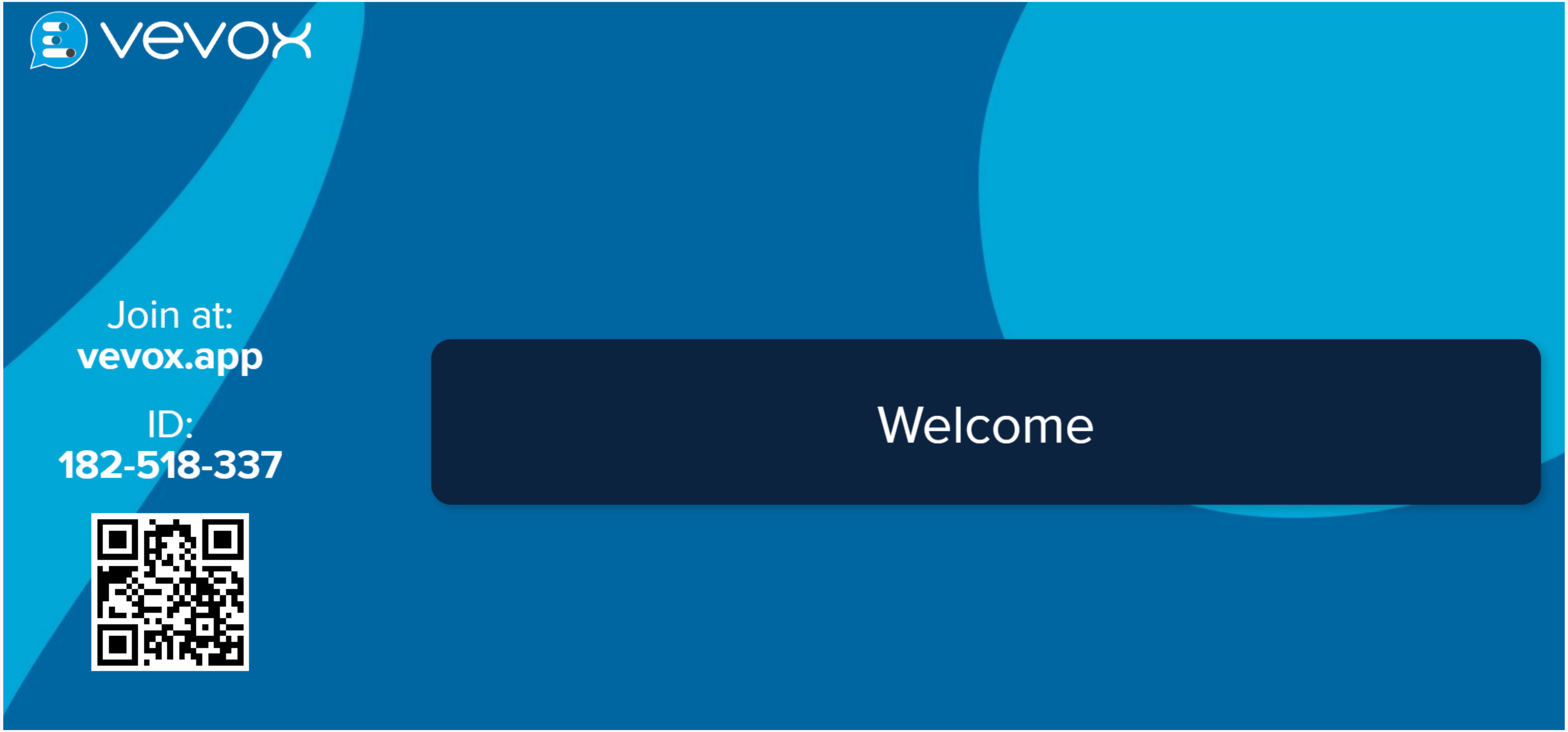
Quiz: are you following?



A note on the non-linear constraint: 23/X1=45

This is indeed a non-linear constraint. X1 is raised to the power of -1.

By rearranging the constraint, you can make it linear: 23-45X1=0

So indeed, many times, it is possible to work with your constraints and make the problems simpler.

But notice a nuance: in the original constraint X1 always had to be positive otherwise the result of the fraction is infinity, whilst in the second version X1 can also be zero. If that's not an issue in your problem, then you can use the linear version.

Break

Why optimization?

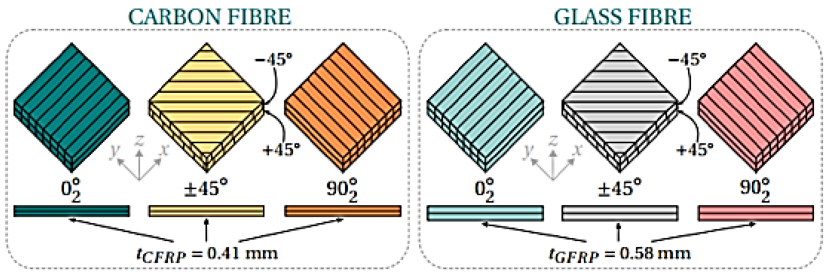
- Many problems require the identification of **the best (or the worst)** solution, decision or design.
- Usually misused. Example, “optimizing the design of a bridge” to refer to the *improvement* of the design, which is not necessary the best design, it is just better.

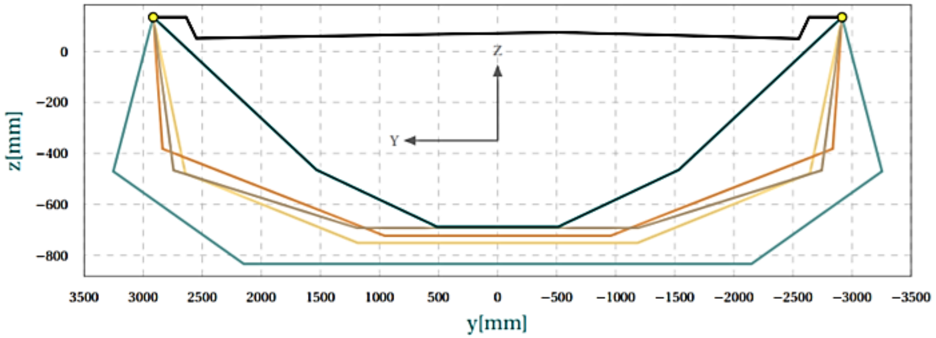
- **Applications**
  - As part of other mathematical tools, such as LS, MLE, ML, etc.
  - Direct applications: Optimal structural design, resource distribution, logistics, etc.

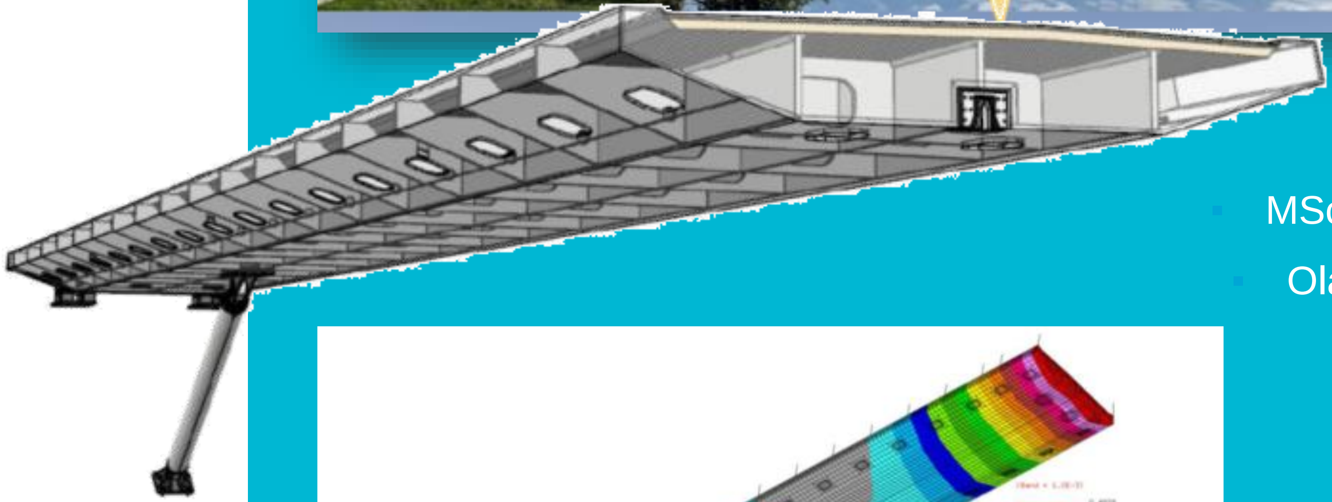
Let’s see some direct applications of optimization problems

Optimizing FRP (Fiber-Reinforced Polymer) bridges

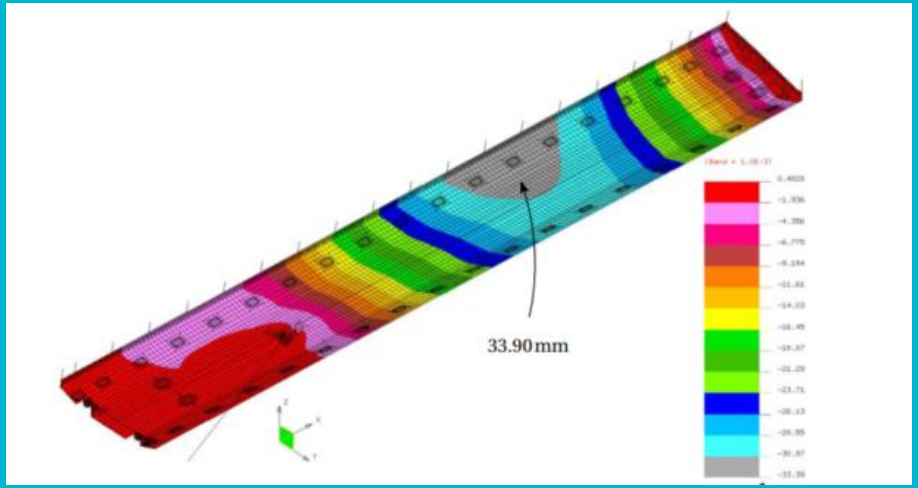
- Goal: to identify the optimal **geometry and plies layout** with consideration of the <u>tradeoff between cost and sustainability</u>. The design must guarantee the structural safety conditions.







- MSc Thesis
- Ola Åsbø



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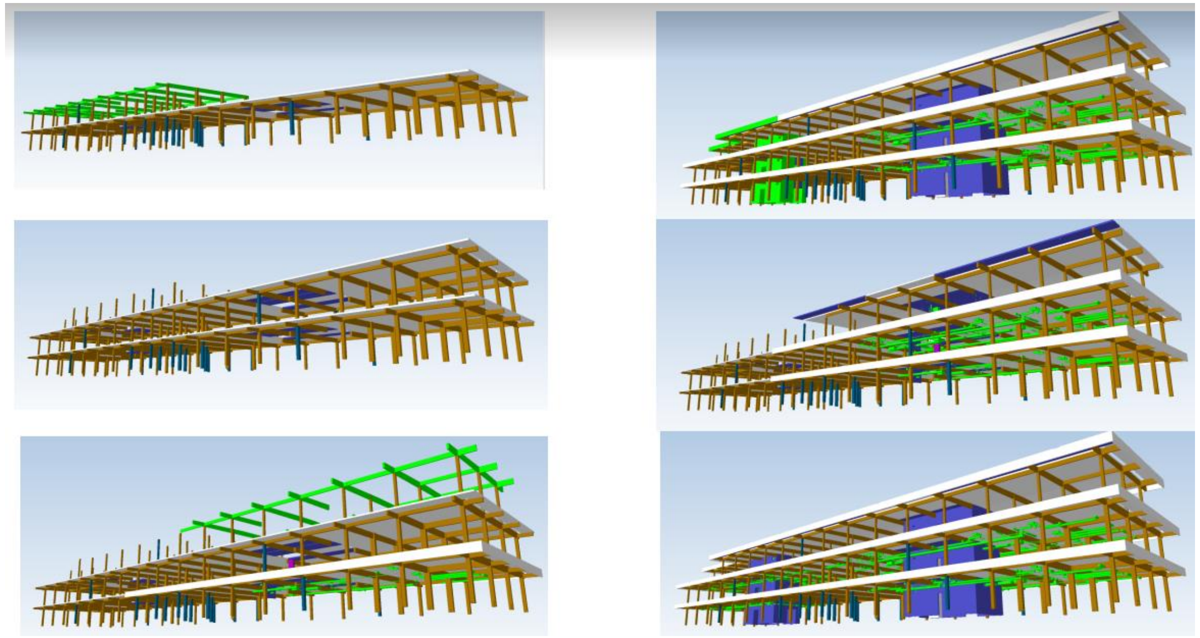
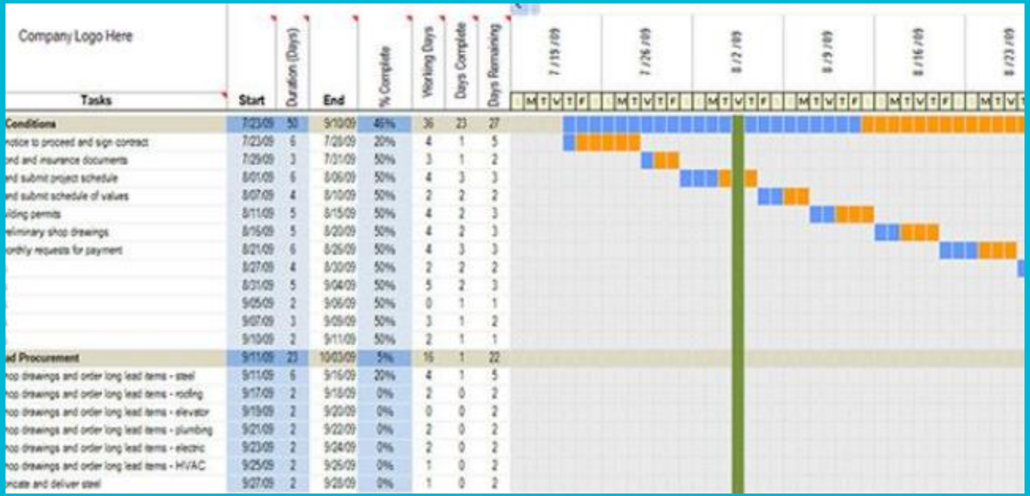
Optimal component level construction schedule

- Goal: to determine the most efficient **process for building components** that have physical interdependencies (fulfilling constructive constraints), with consideration of the tradeoff between the cost and construction duration.

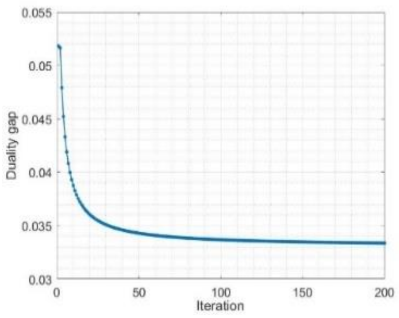


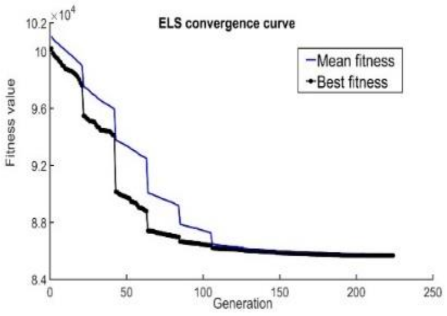
MSc Thesis - Xinzhi Jiang

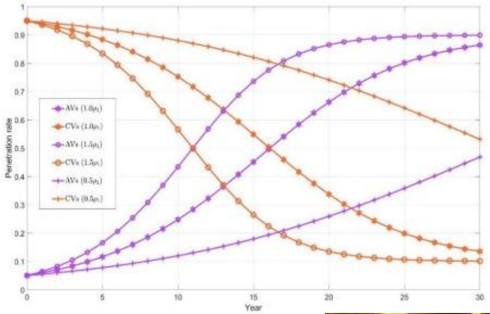


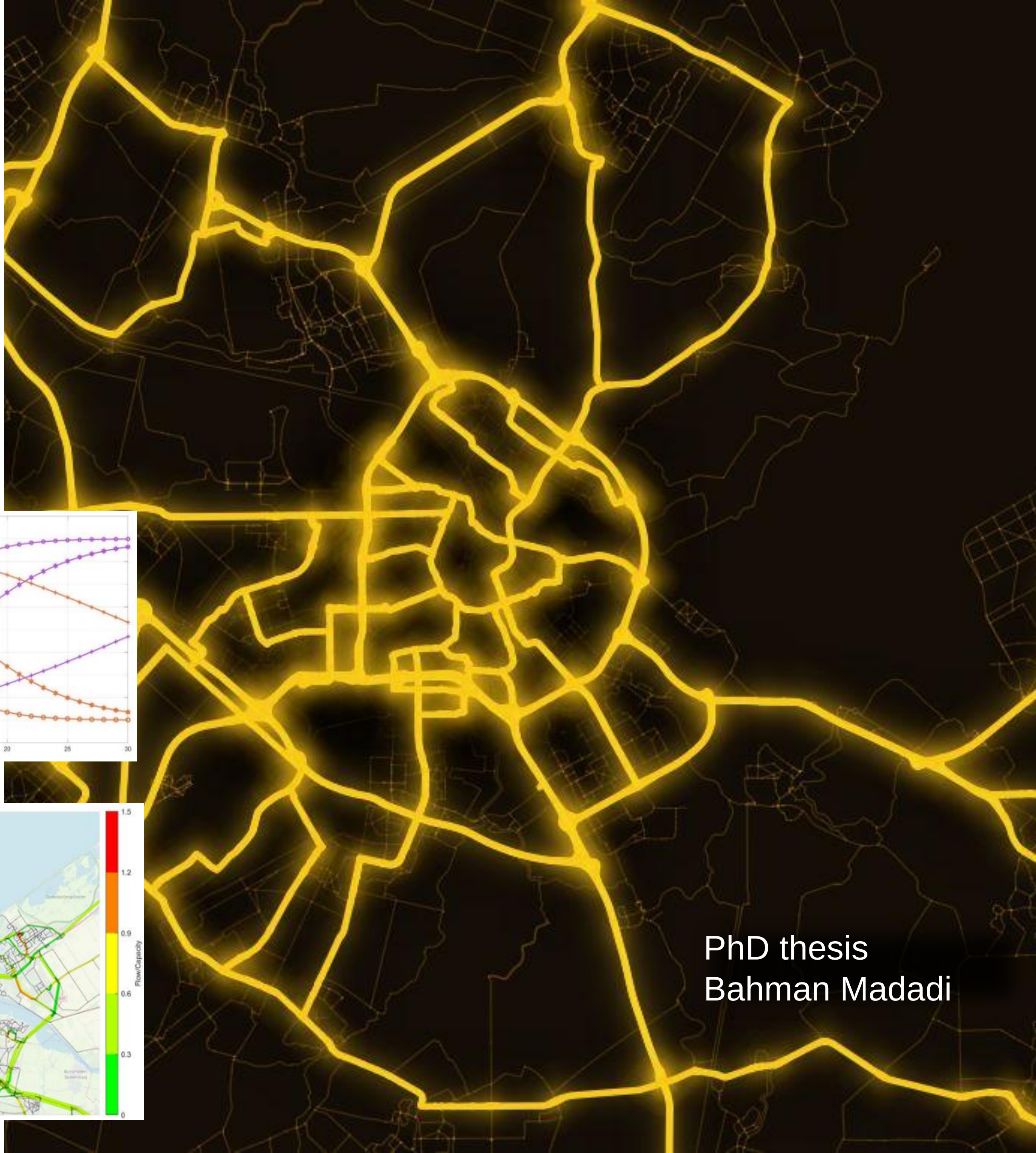
Designing road networks for automated vehicles

- Goal: To identify the best **location and time for deploying enhanced roads** for automated vehicles, <u>minimizing deployment cost and maximizing efficiency and safety</u> given the uncertain evolution path of automated driving technology and the travelers’ mode and route choice behavior.





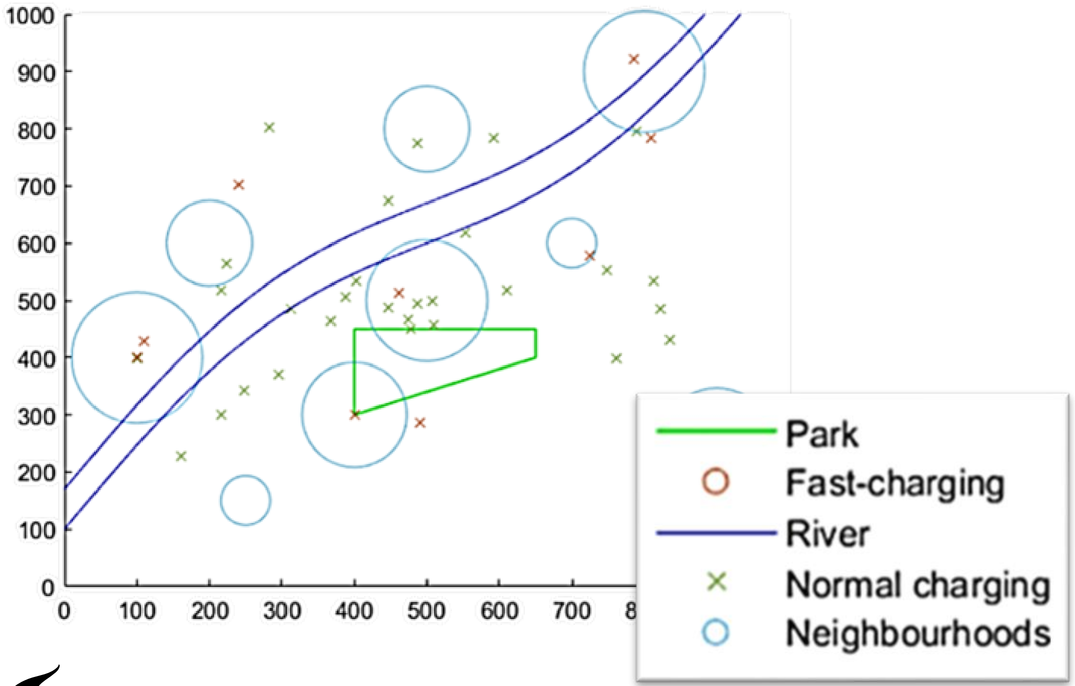


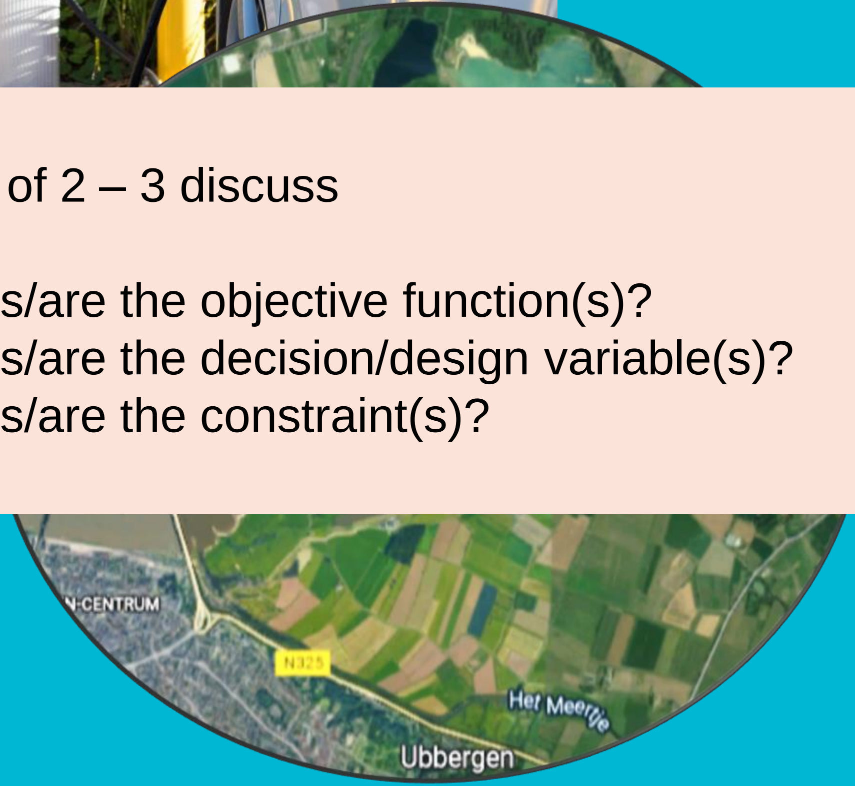


PhD thesis
Bahman Madadi

Optimization of electric vehicles charging station locations

- Goal: to identify the optimal distribution of charging stations that maximizes the utility accounting for the population density, existence of other charging stations, and other urban elements (e.g., rivers, parks…).





In groups of 2 – 3 discuss

- ❑ What is/are the objective function(s)?
- ❑ What is/are the decision/design variable(s)?
- ❑ What is/are the constraint(s)?

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Optimization of electric vehicles charging station locations

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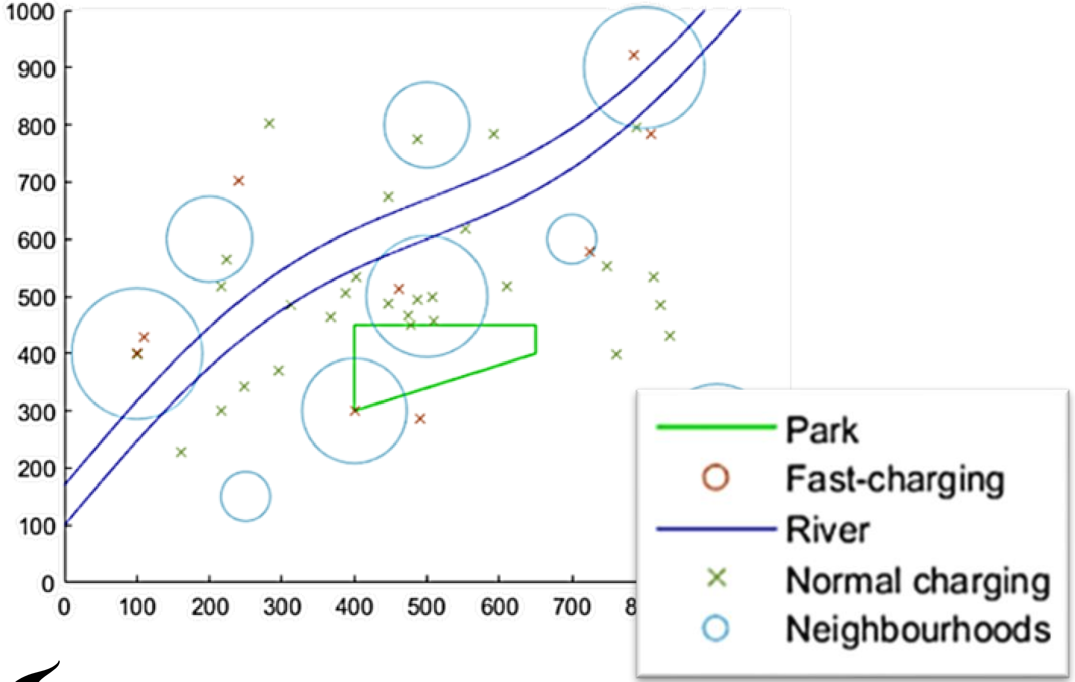

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What is the objective function?

✓ Maximize the utility (e.g., user accessibility, service coverage, etc.)

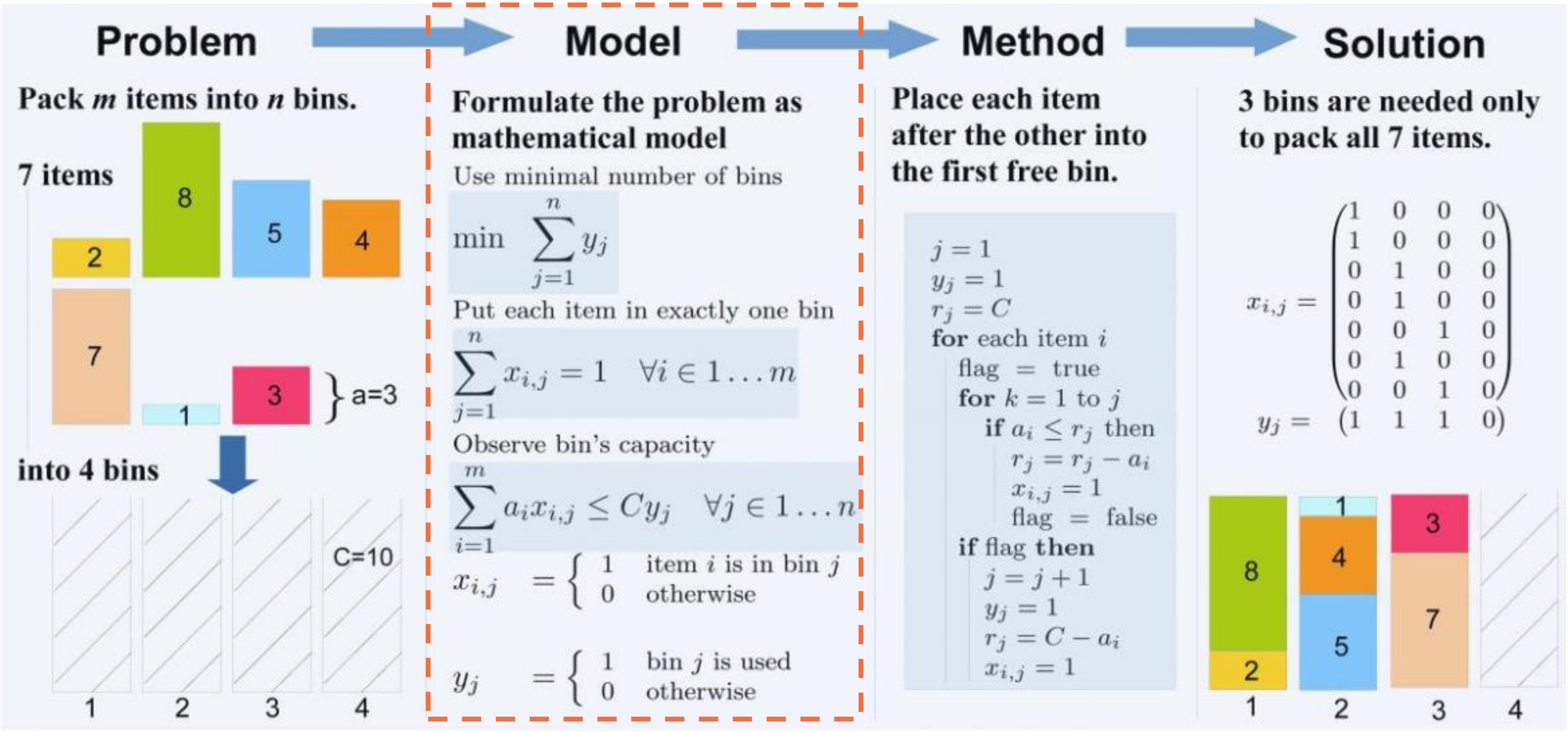
What is the decision/design variable?

✓ Distribution of charging stations (e.g., coordinates, 0/1 grid, number and average distance, etc.)

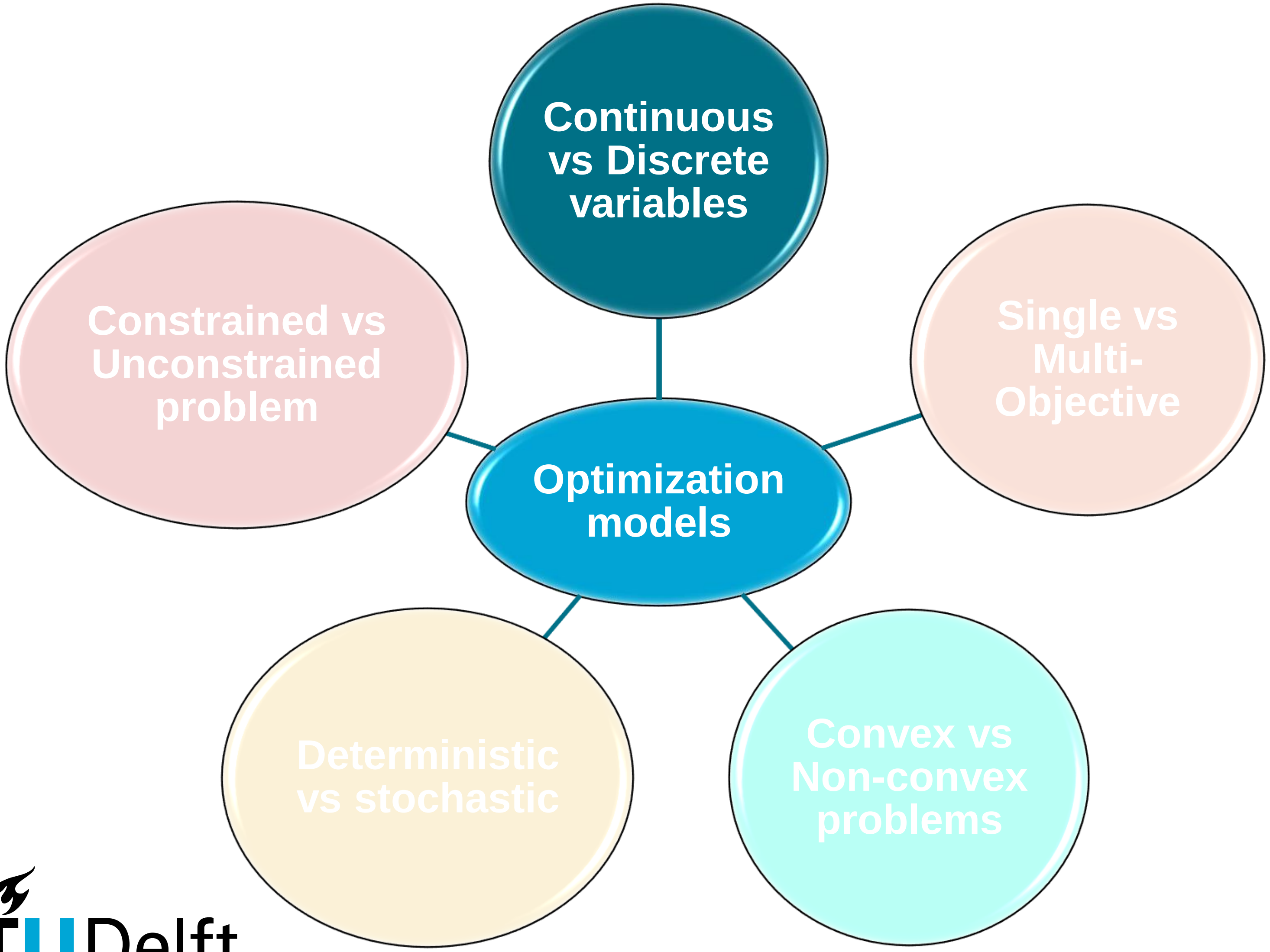
What are the constraints?

✓ Minimum charger-citizen ratio
✓ Minimum distance to other charging stations
✓ Maximum distance to other charging stations
✓ No chargers in a park, in a river, etc.
✓ …

What is included under the concept of optimization?



Optimization Models. Taxonomy



- **Continuous variables**:

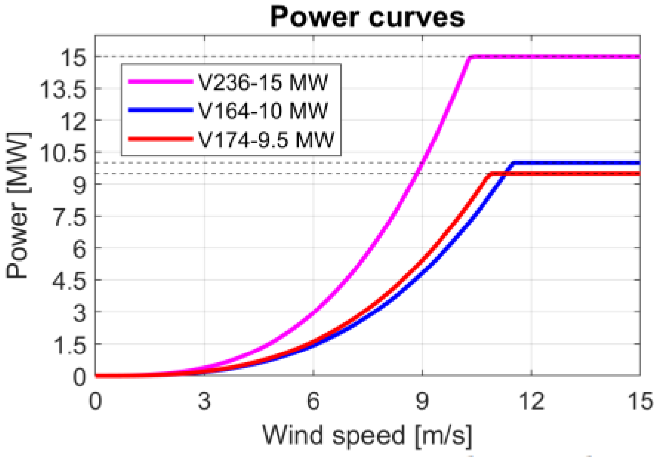
  time, distances, physical properties, etc.

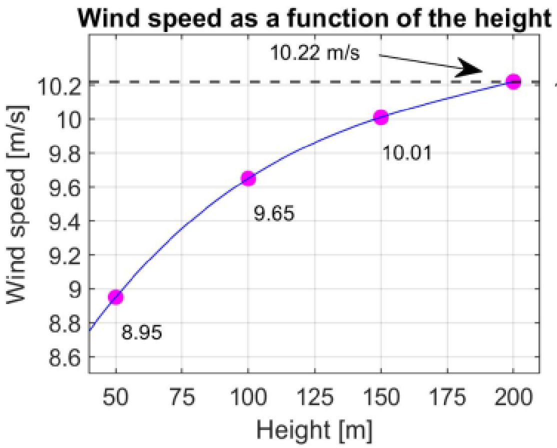
- **Discrete variables**:

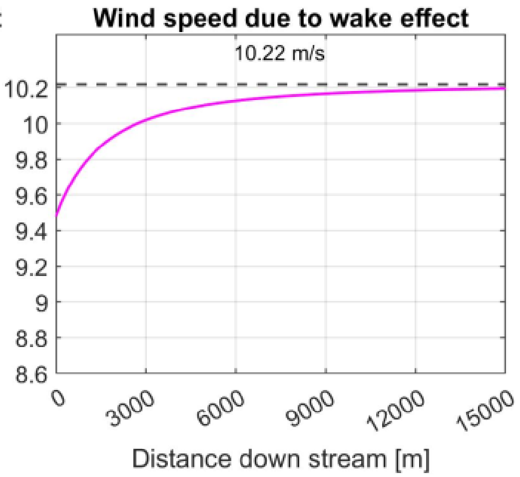
  number of wind turbines, decisions such as doing something or not, type of materials, etc.

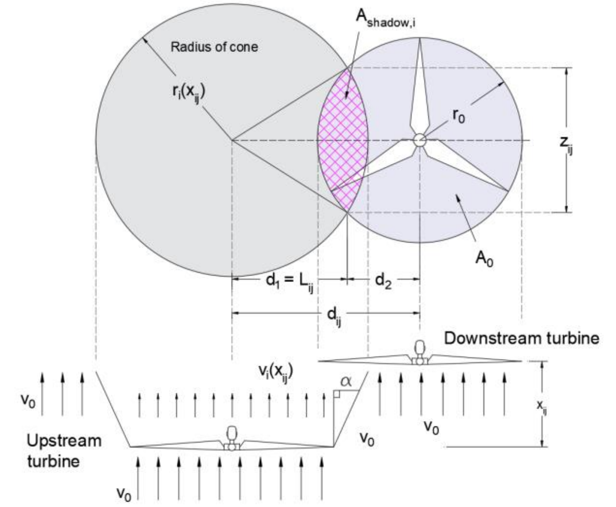
Optimizing the layout of the offshore wind farms in Norway

- Goal: Determine the optimal **layout of wind turbines** to produce the <u>highest annual energy production with the minimum cost</u>. Considering different types of turbines, their power curves, physical characteristics, wake effect, etc.





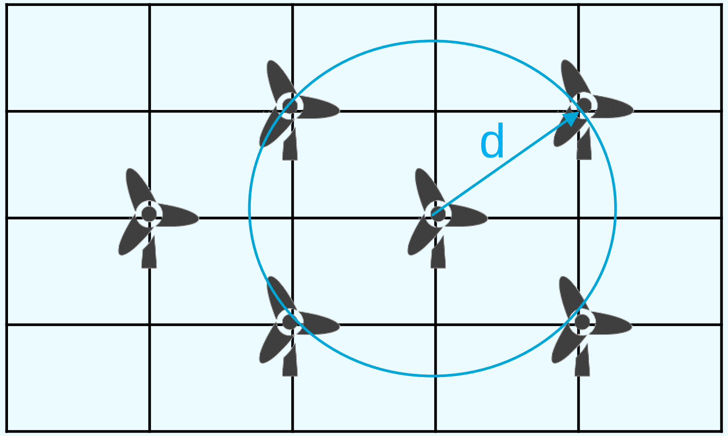




CME4501 Engineering Systems Optimization - Final project

J. Aulbers, O. Åsbø & I. Timori

Optimal Wind Turbine (WT) Farm



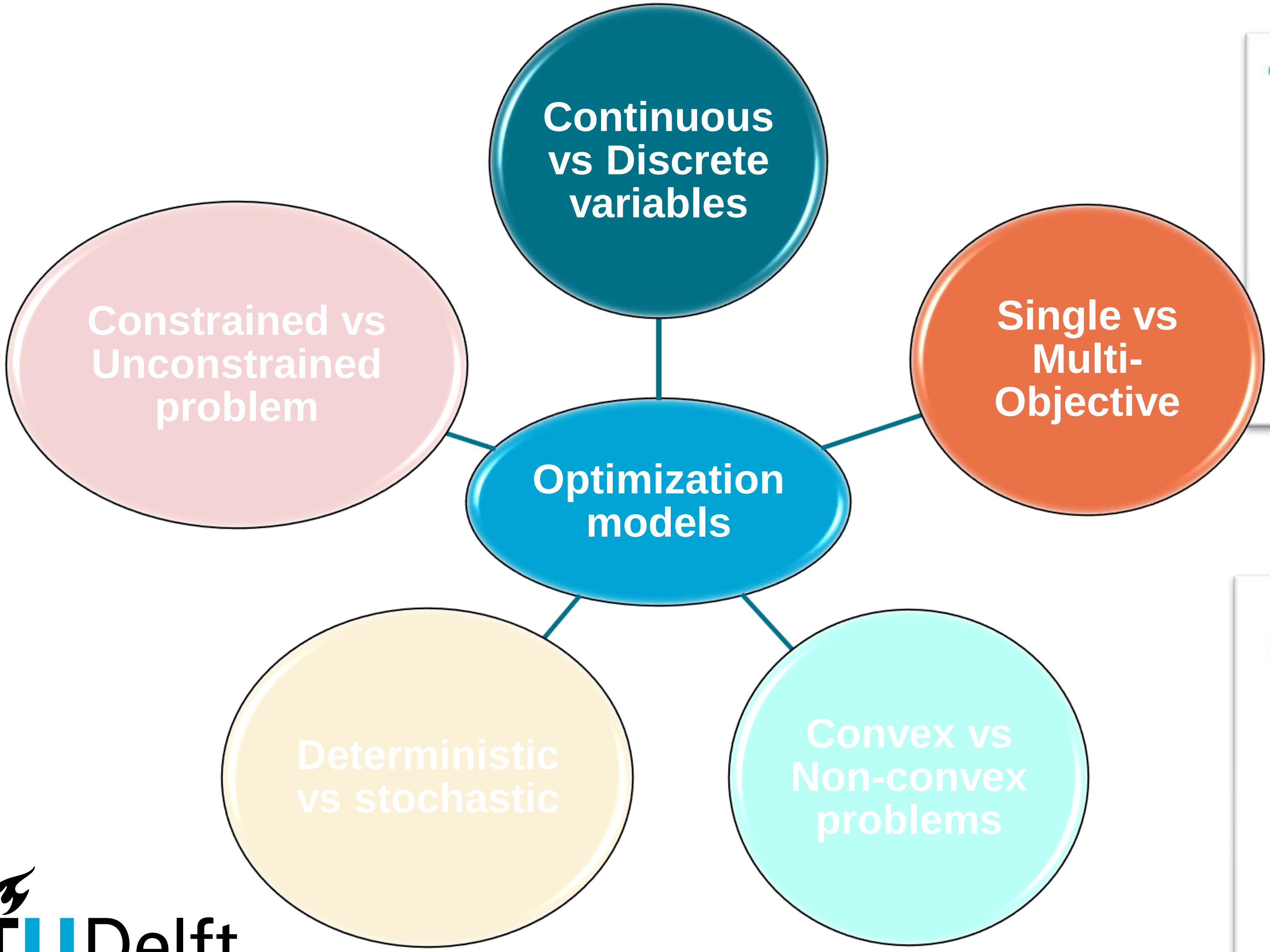
Discrete variables

Continuous variables

Decision variables:

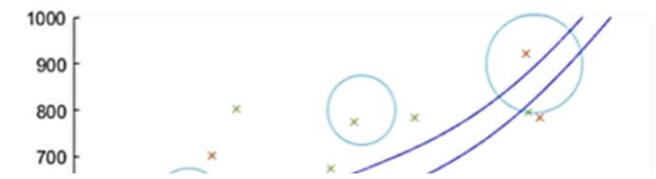
- **n**: number of WTs (n is between 10 and 50).
- **d**: the closest distance between two WTs (d is between 15 and 100 m).

Optimization Models. Taxonomy



- **Single objective:**

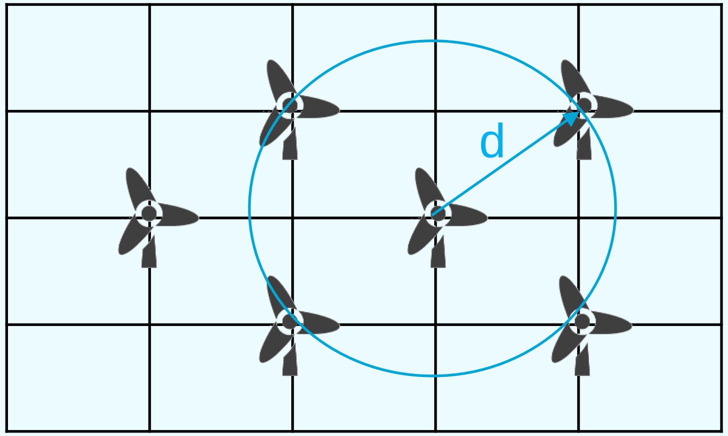
  - Goal: to identify the optimal **distribution of charging stations** that maximizes the utility accounting for the population density, existence of other charging stations, and other urban elements (e.g., rivers, parks…).



- **Multi-objective:**

  - Goal: Determine the optimal **layout of wind turbines** to produce the highest annual energy production with the minimum cost. Considering different types of turbines, their power curves, physical characteristics, wake effect, etc.

Optimal Wind Turbine (WT) Farm



Decision variables:

- **n**: number of WTs, [10,50]
- **d**: the closest distance between two WTs [15,100]

Single objectives

Multi-objective

Objective function:

- Maximize the annual production:

  $\boldsymbol{Max_{n,d}\ \ n.P_{unit}(d)}$

  with $P_{unit}(d)$ being the energy production of 1 WT that depends on the distance between WTs

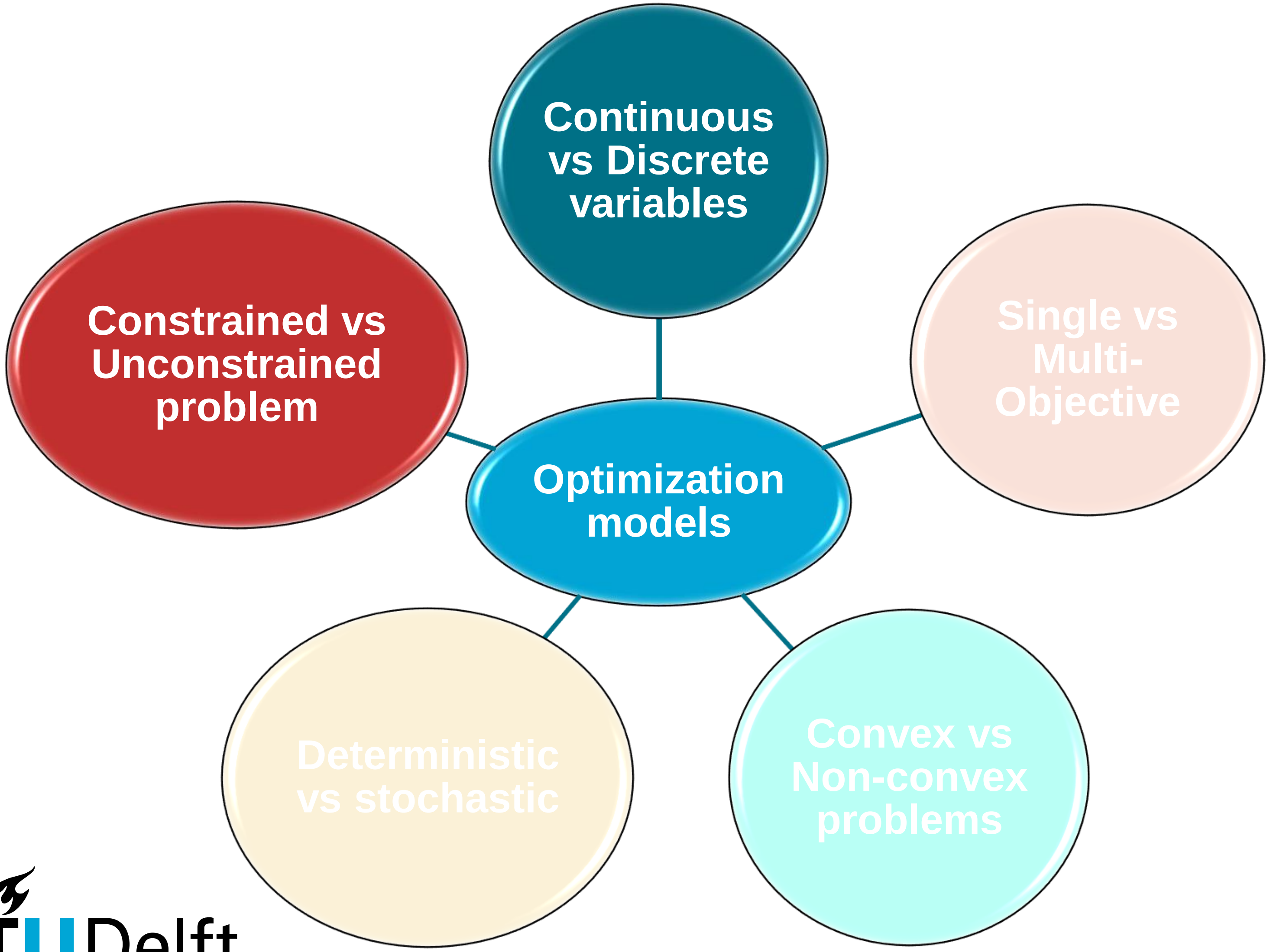
- Minimise the annual maintenance cost:

  $\boldsymbol{Min_{n,d}\ \ n.C_m}$

  with C_m being the annual maintenance cost of 1 WT

- Both objectives: $\boldsymbol{Min_{n,d}\ \{-n.P_{unit}(d);\ n.C_{unit}\}}$

Optimization Models. Taxonomy



- **Unconstrained problem:** the solution space is not bounded. All the configurations are possible candidates for being optimal.

- **Constrained problem:** the solution space is bounded.
  - <u>Feasible region</u>: Only a set of solutions are possible candidates.
  - <u>Unfeasible region</u>: There is not a possible solution fulfilling all the constraints.

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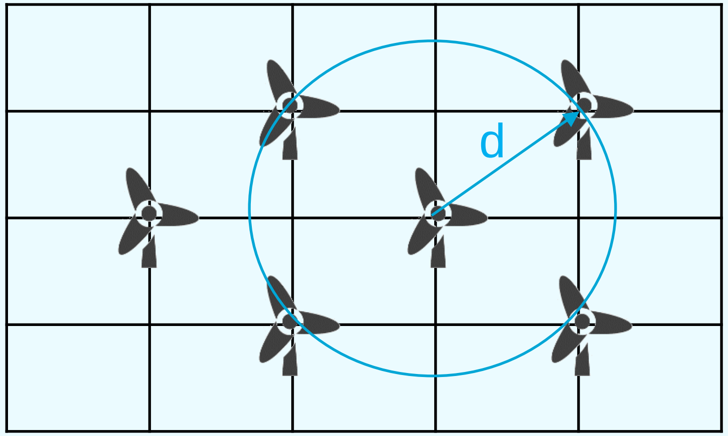
Optimal Wind Turbine (WT) Farm



Decision variables:

- **n**: number of WTs, [10,50]
- **d**: the closest distance between two WTs [15,100]

Objective function:

- Maximize the annual production: $\boldsymbol{Max_{n,d}\ n.P_{unit}(d)}$

Constrained problem

If the construction (C) or maintenance costs (C_m) of a WT is larger than 5M or 0,07M EURO respectively, there is no possible solution. In such a case, we have an unfeasible problem.

Constraints:

- Limited construction budget of 50M EURO:

  $\boldsymbol{n\,C \leq 50M}$

  with C being the construction cost of 1 WT

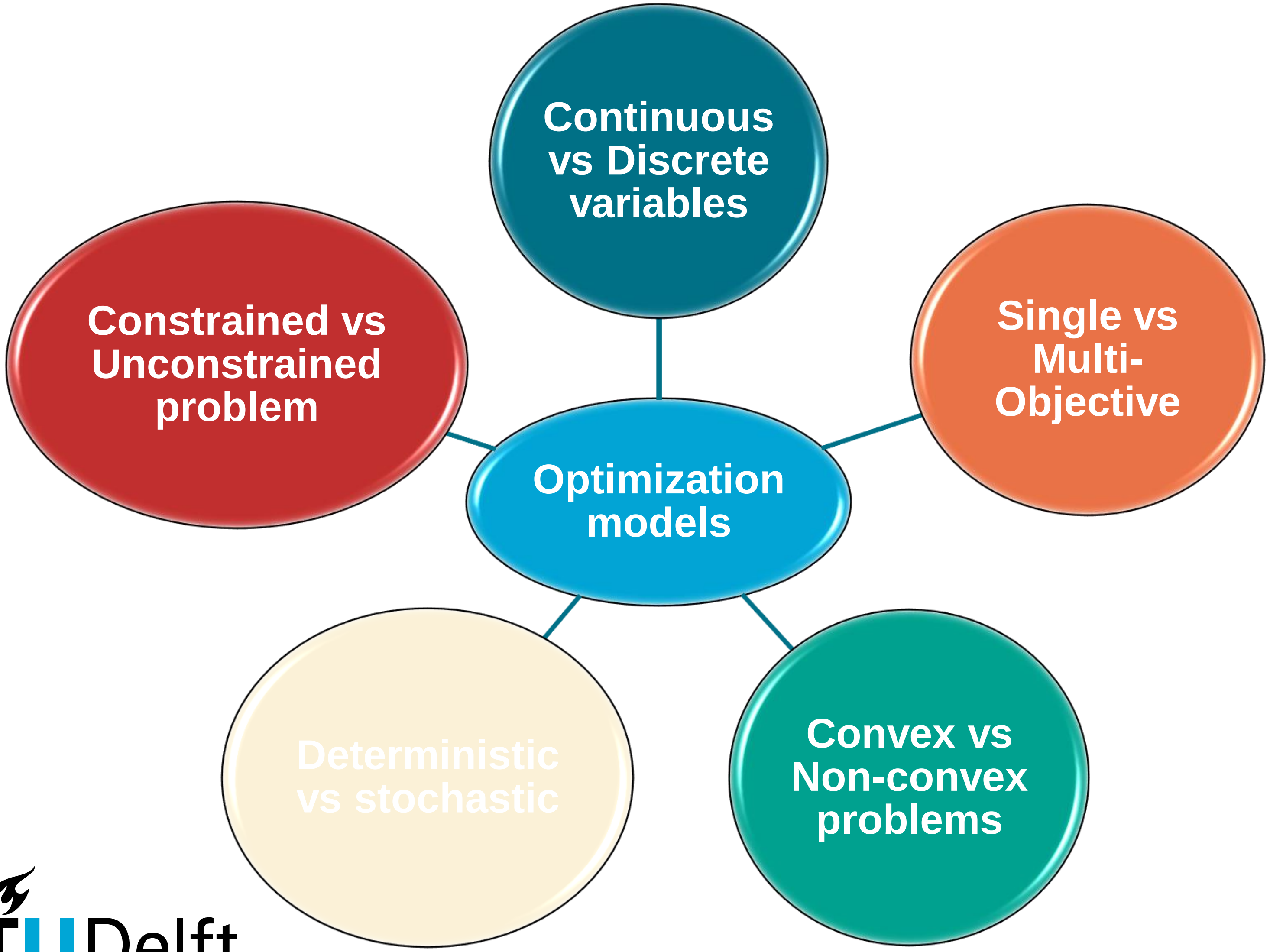
- Limited annual maintenance cost of 0,7M EURO:

  $\boldsymbol{n\,C_m \leq 0,7M}$

  with C_m being the annual maintenance cost of 1 WT

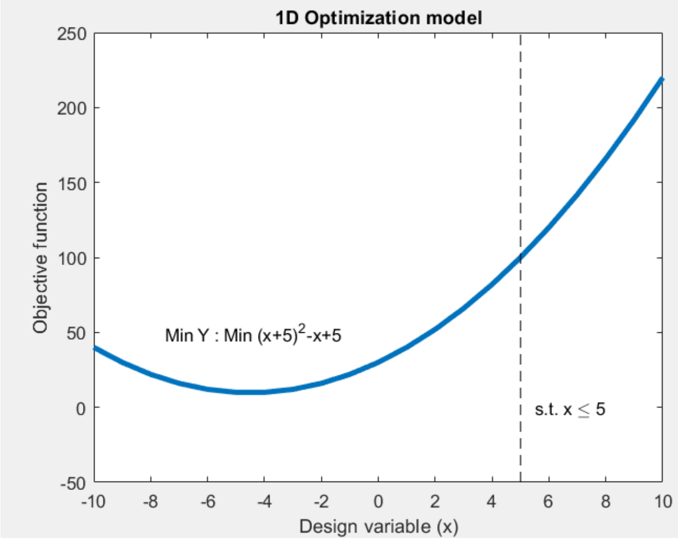
- …

Optimization Models. Taxonomy

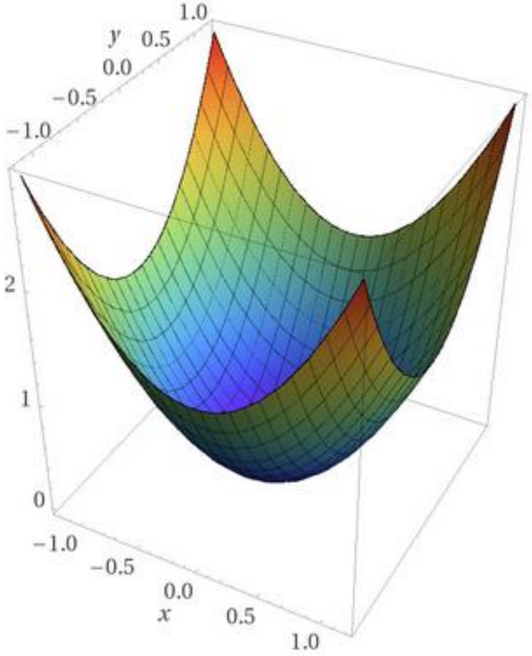


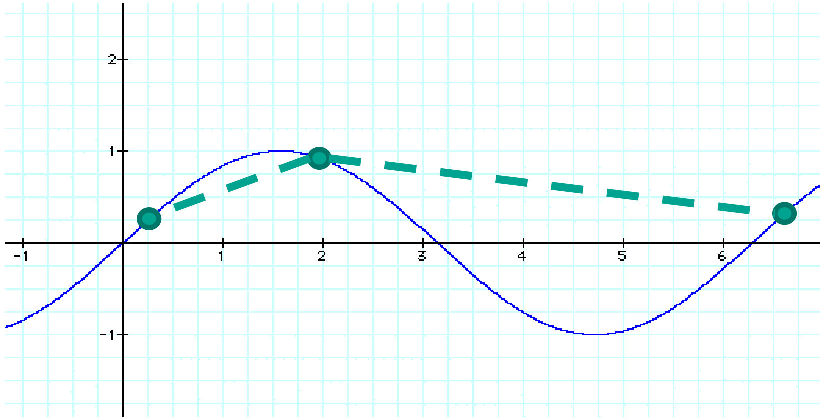
Convex problems

A convex optimization problem is a problem where all the constraints and the objective are convex functions

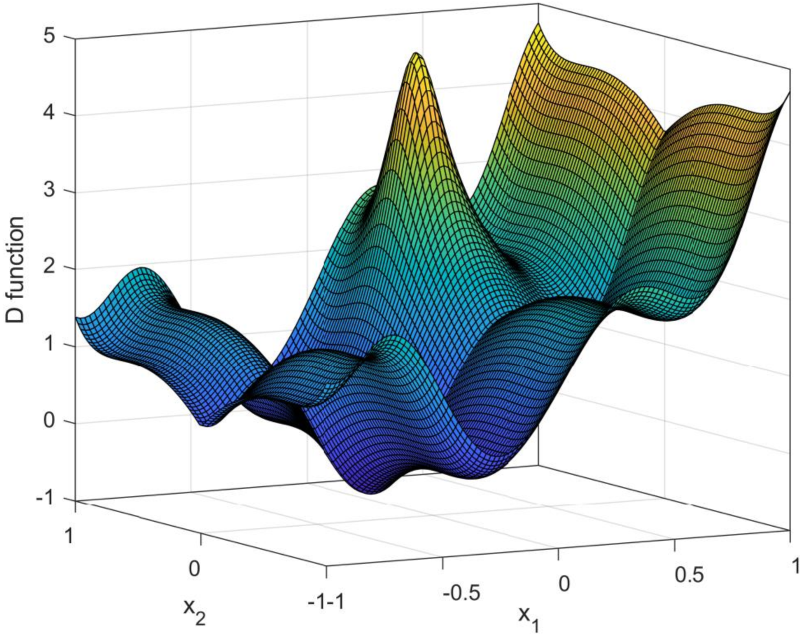


Convex function

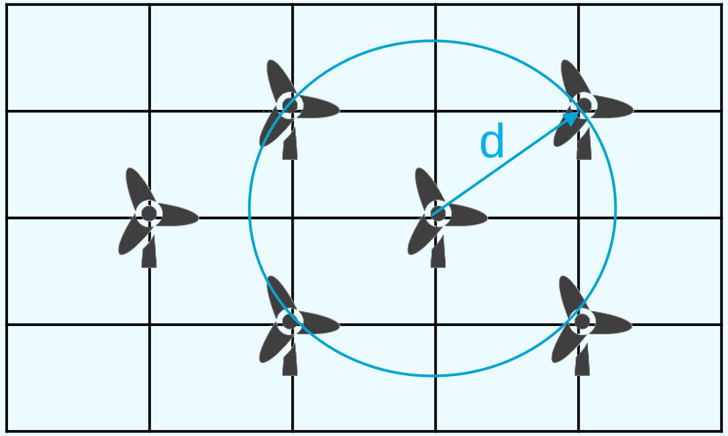




Non-convex function



Optimal Wind Turbine (WT) Farm



Decision variables:

- **n**: number of WTs, [10,50]
- **d**: the closest distance between two WTs [15,100]

Objective function:

- Maximize the annual production: $\boldsymbol{Max_{n,d}\ n.P_{unit}(d)}$

Constraints:

- $\boldsymbol{n\,C \leq 50M}$
- $\boldsymbol{n\,C_m \leq 0{,}7M}$

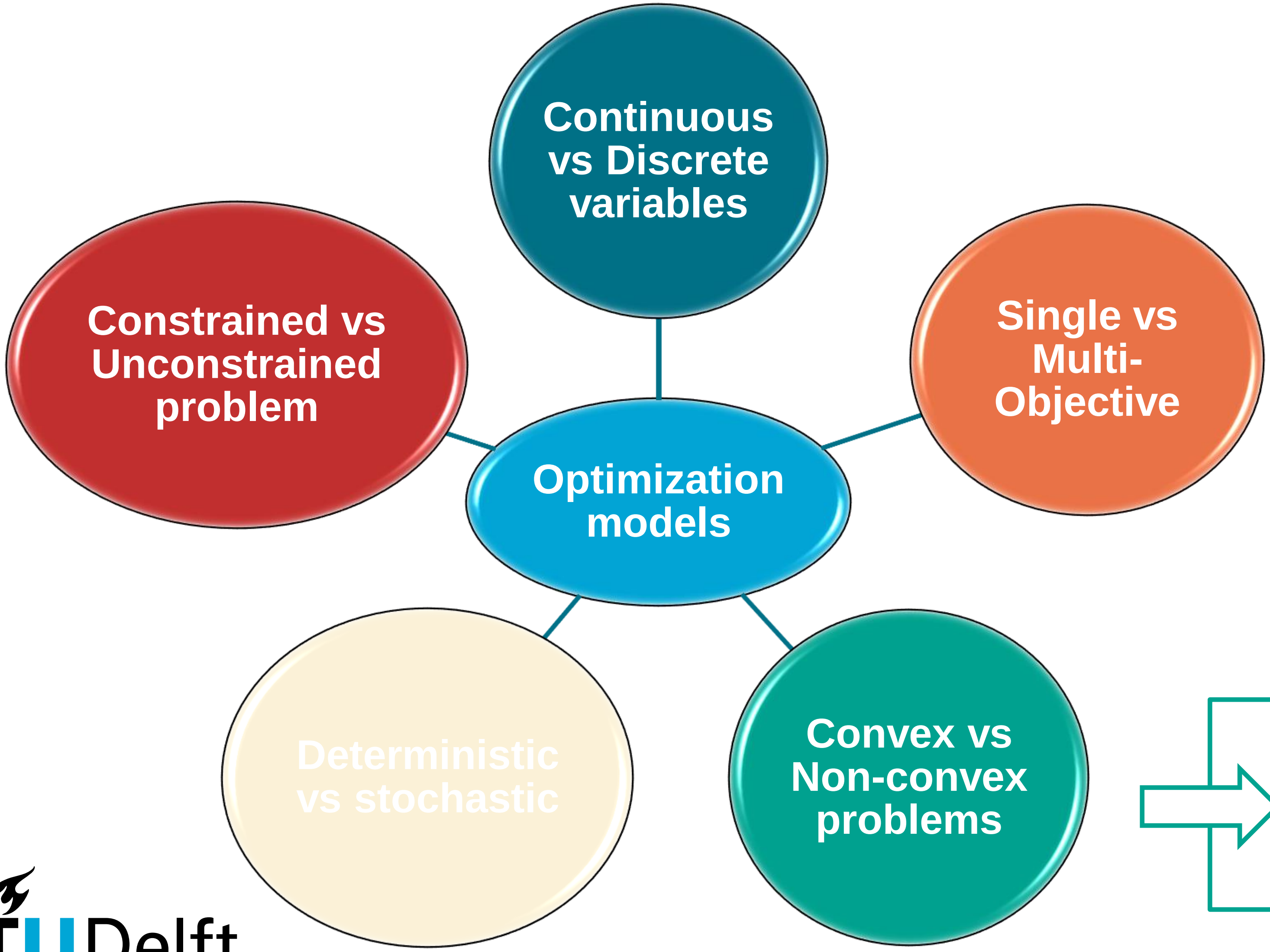
Linear = convex

Even if $\boldsymbol{P_{unit}(d)}$ were linear, the objective function would involve the product of two variables, thus, it is non-convex.

- ➢ If both, the objective function and all constraints are convex, the problem is convex.
- ➢ Linear and quadratic functions are convex.

✓ This optimization problem is NON-CONVEX because the objective function is non-convex

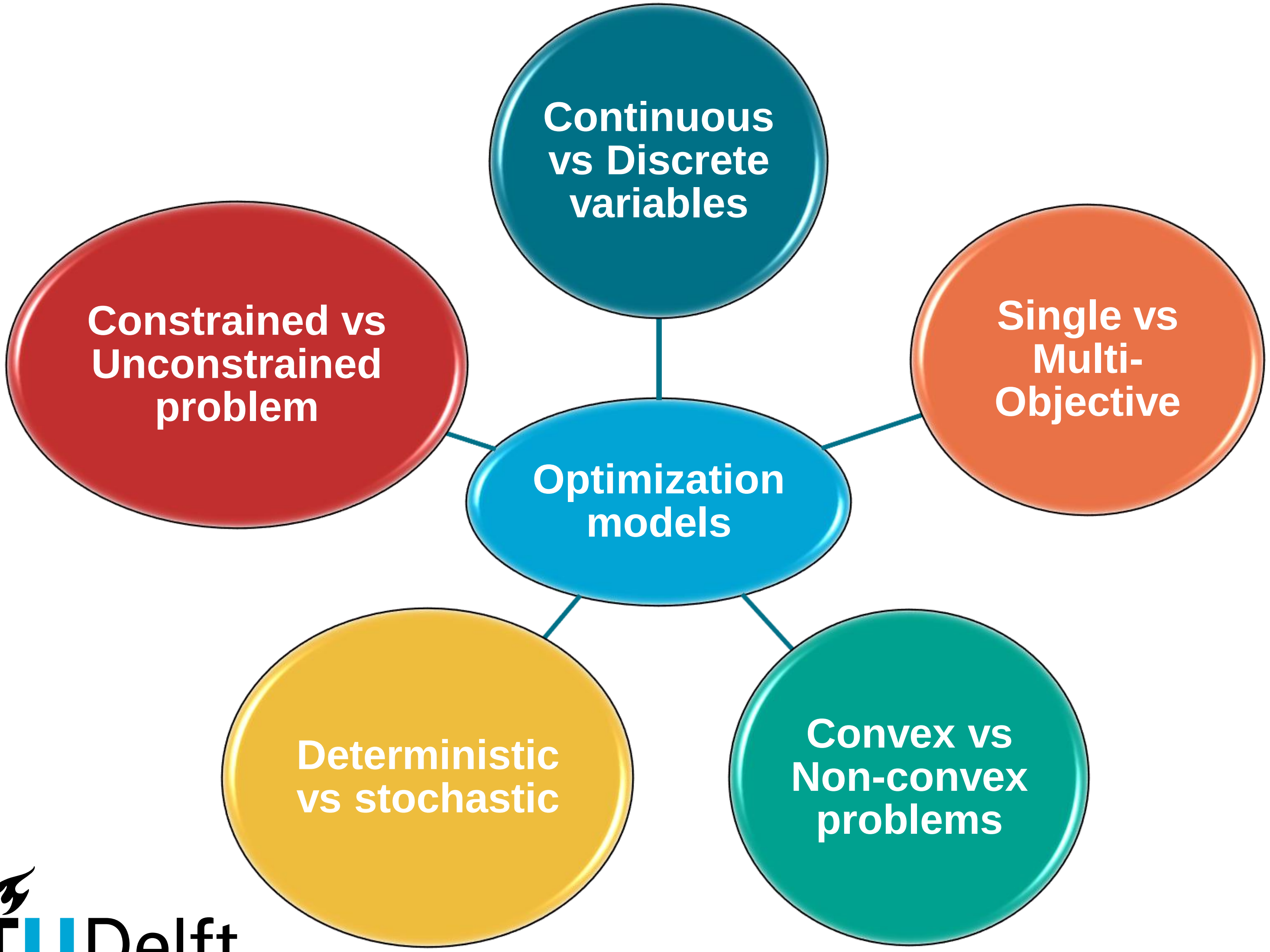
Optimization Models. Taxonomy



Common convex problem:

- Linear Programming (LP) ✓

Optimization Models. Taxonomy



- **Deterministic optimization**: all the parameters of the optimization problem are deterministic. There is not variability in the problem definition.

- **Stochastic optimization**: The definition of optimization problem presents variability or uncertainty. The optimal solution of a possible scenario is not necessary the optimal solution of another possible scenario.

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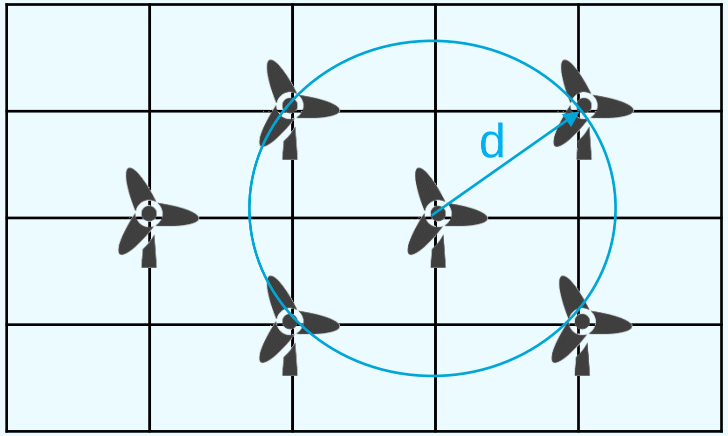
Optimal Wind Turbine (WT) Farm



Decision variables:

- **n**: number of WTs, [10,50]
- **d**: the closest distance between two WTs [15,100]

Objective function:

- Maximize the annual production: $\boldsymbol{Max_{n,d}\ n.P_{unit}(d)}$

Constraints:

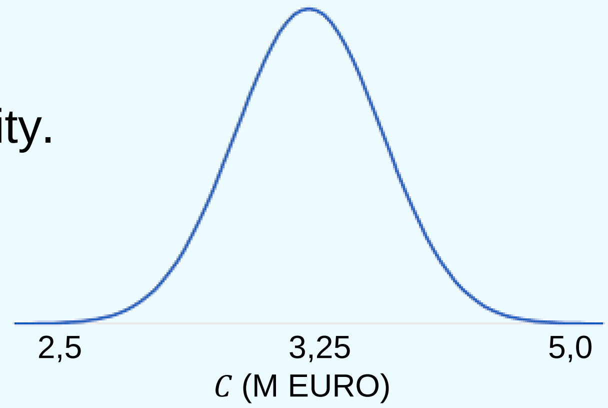
- $\boldsymbol{n\,C \le 50M}$
- $\boldsymbol{n\,C_m \le 0,7M}$

In case the construction (C) or/and maintenance costs (C_m) present variability.
Then, the constraints can be expressed as

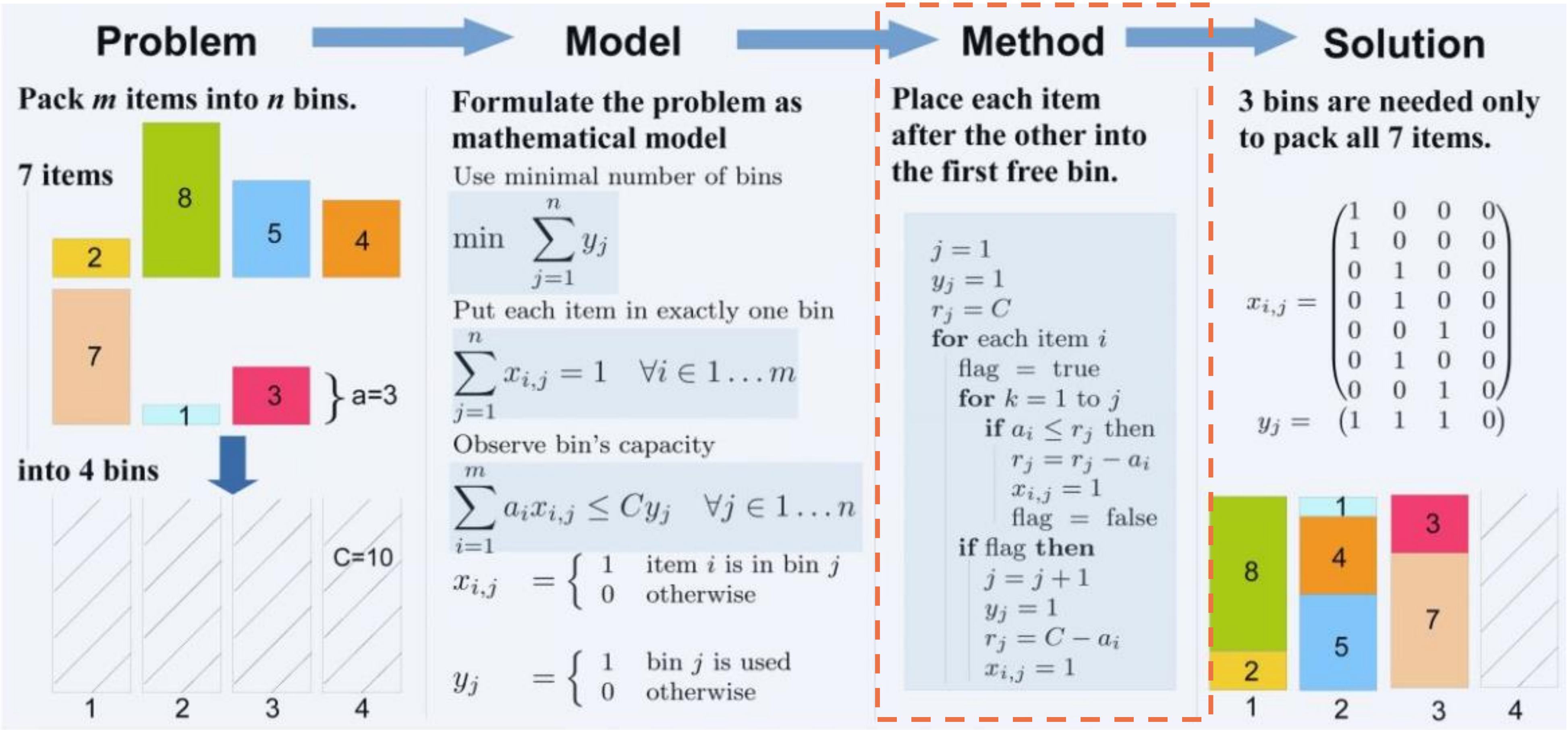
$$\boldsymbol{Prob(n\,C \le 50M) \ge 0,90}$$
$$\boldsymbol{Prob(n\,C_m \le 0,7M) \ge 0,95}$$

Deterministic

Stochastic



What is included under the concept of optimization?



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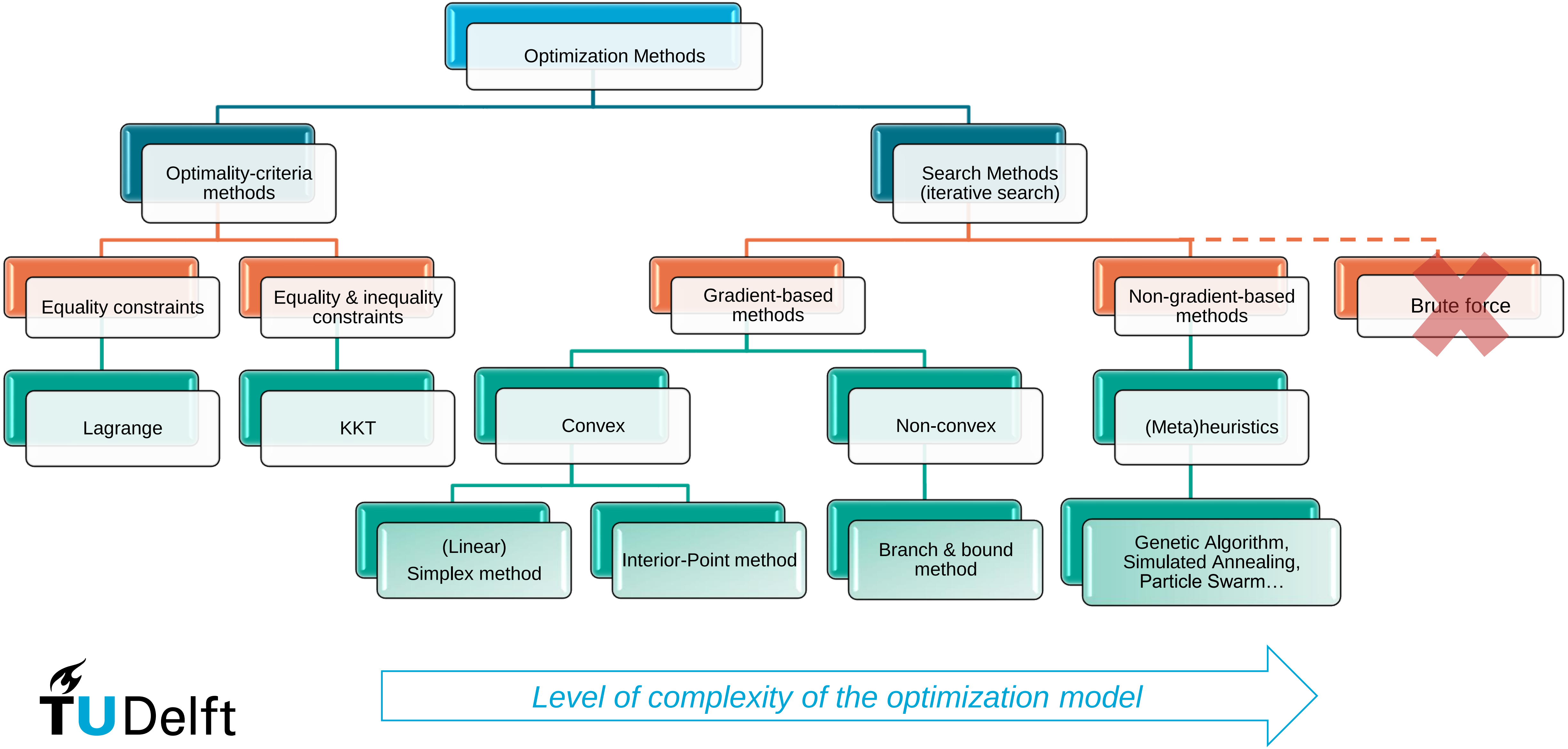
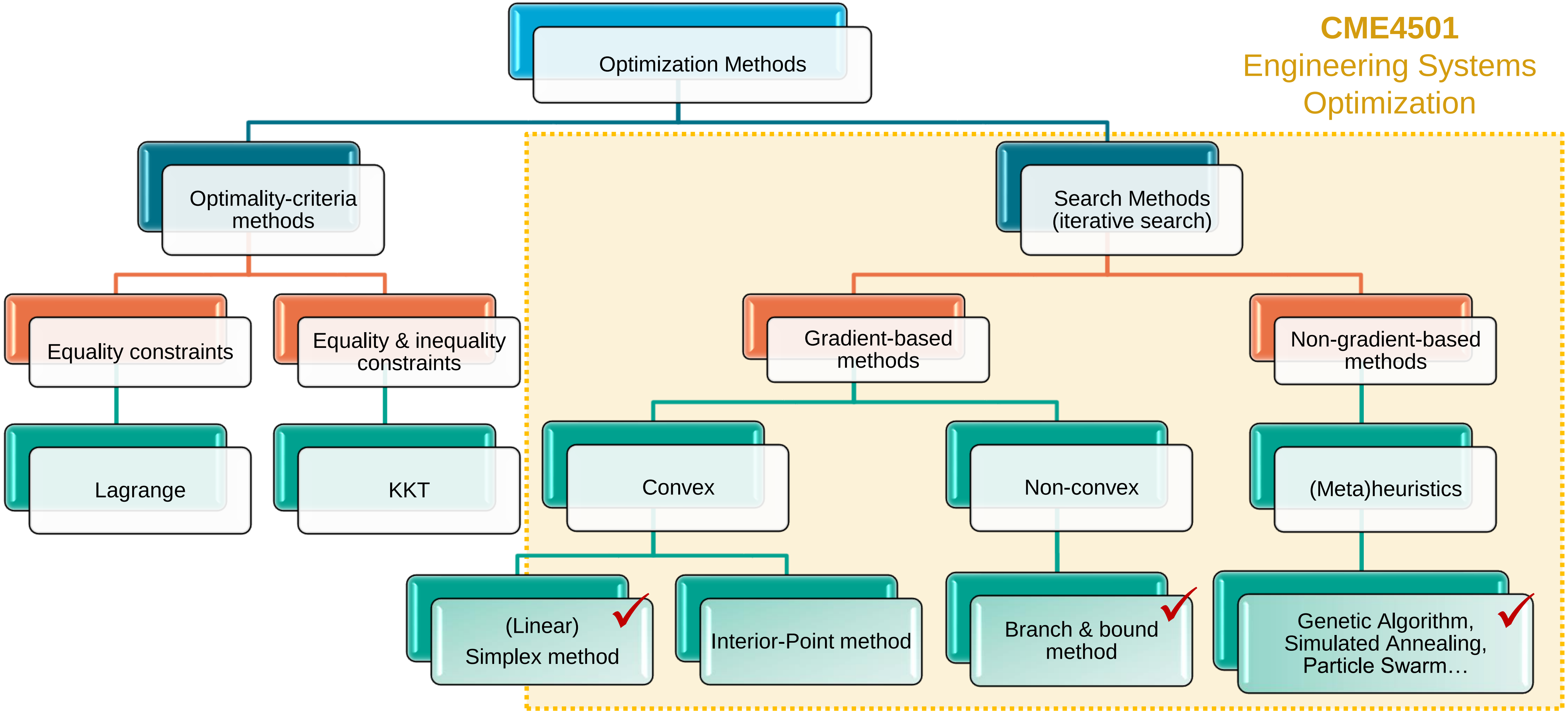
Optimization Methods. Taxonomy



Optimization Methods. Taxonomy



Suggested schedule

Monday	Tuesday	Wednesday	Thursday	Friday
Optimization Kick-off	Q&A Session	Workshop Session *Integer programming problem (Planet versus Profit)*	Q&A Session	Project Session *Road network problem*
Suggested progress:	Suggested progress:	Suggested progress:	Suggested progress:	
Read Sec 5.1-5.3 **Sec 5.4** **Video 1 and 2** Sand and clay problem formulation **Sec 5.5** **Video 3** Augmented form of math problem	**Sec 5.6** **Video 4** SIMPLEX Method **Sec 5.7** **Video 5** Integer Programming	After the workshop: **Sec 5.9** **Video 6** Genetic algorithm	**Sec 5.11** Road network design problem (this introduces the project for Friday)	

Questions!