

Sinusoidal force field

- 1) Given $F = F_0 \sin \frac{x}{L}$

If conservative, a potential does exist. So, if we can find the potential, then we know F is conservative

$$V = - \int_{\frac{\pi}{2}L}^x F dx' = F_0 L \left[\cos \frac{x'}{L} \right]_{\frac{\pi}{2}L}^x = F_0 L \cos \frac{x}{L}$$

So, we found a potential and thus F is conservative.

- 2) The particle is initially at $x=0$. At this point the force $F=0$. Consequently, the particle will not move. However, the slightest push will have it move away from the point $x=0$. As soon as the particle is out of $x=0$ it does experience a non-zero force and will move.

- 3) The total energy of the particle is constant. This means that

$$\frac{1}{2}mv^2 + F_0 L \cos \frac{x}{L} = E_0$$

Initially, $v=0$ (that is so small that for all practical calculations the initial kinetic energy can be taken as zero). Thus

$$E_0 = F_0 L \cos 0 = F_0 L$$

and

$$v = \sqrt{\frac{2}{m} \left(F_0 L - F_0 L \cos \frac{x}{L} \right)}$$

This has a maximum when the cos-term has a minimum. This happens at

$$x = (\pi \pm 2k\pi)L$$

There

$$v_{max} = 2 \sqrt{\frac{F_0 L}{m}}$$