

No sketch needed: this is a purely mathematical exercise.

1)

$$\text{a) } \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0 \quad \text{thus } F \text{ is conservative and a potential exists}$$

$$\text{b) } \vec{F} = -\vec{\nabla}V \rightarrow$$

$$\frac{\partial}{\partial x}V = -x \rightarrow V = -\frac{1}{2}x^2 + f(y, z)$$

$$\frac{\partial}{\partial y}V = -y \rightarrow V = -\frac{1}{2}y^2 + g(x, z)$$

$$\frac{\partial}{\partial z}V = -z \rightarrow V = -\frac{1}{2}z^2 + h(x, y)$$

$$\text{Thus, we can take: } V(x, y, z) = -\frac{1}{2}x^2 - \frac{1}{2}y^2 - \frac{1}{2}z^2 + C$$

With C any constant we like.

2)

$$\text{a) } \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & x & z \end{vmatrix} = 0\hat{x} - 0\hat{y} + \left(\frac{\partial}{\partial x}x - \frac{\partial}{\partial y}y\right)\hat{z} = 0$$

thus F is conservative and a potential exists.

$$\text{b) } \vec{F} = -\vec{\nabla}V \rightarrow$$

$$\frac{\partial}{\partial x}V = -y \rightarrow V = -xy + f(y, z)$$

$$\frac{\partial}{\partial y}V = -x \rightarrow V = -xy + g(x, z)$$

$$\frac{\partial}{\partial z}V = -z \rightarrow V = -\frac{1}{2}z^2 + h(x, y)$$

$$\text{Thus, we can take: } V(x, y, z) = -xy - \frac{1}{2}z^2 + C$$

With C any constant we like.