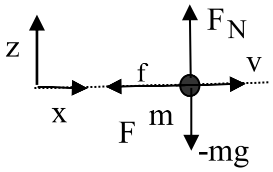
Exercise Hockey Puck on ice



a) sketch: x,z axis ->
in z-direction: $F_N - mg = 0$ no nett force, nothing interesting happens.
in x-direction: 1 force: friction $F_f = -\mu mg$ in the negative x-direction
equation of motion:

$$F_f = ma_x \rightarrow m\frac{dv}{dt} = F_f = -\mu mg$$

1st order diff.eq. for v: 1 initial condition needed: at t=0, v=v_0

b) $\frac{dv}{dt} = -\mu g \rightarrow v(t) = -\mu gt + C_1 \; since \; g = const$
i.c.: $t =\rightarrow v = v_0 \Rightarrow v_0 = 0 + C_1$
solution: $v(t) = v_0 - \mu gt$

Solve for position x(t):
$\frac{dx}{dt} = v = v_0 - \mu gt \rightarrow x(t) = v_0 t - \frac{1}{2}\mu gt^2 + C_2$ as both v_0 and g are const.

i.c. $t = 0 \rightarrow x = 0$ which is our own choice for the z-axis. This gives for C_2:

$$0 = 0 - 0 + C_2 \rightarrow C_2 = 0$$

thus the solution of the trajectory of the object is:
$x(t) = v_0 t - \frac{1}{2}\mu gt^2$

c) Work: from x=0 to where puck stops.
puck stops at t^*
$v(t^*) = 0 \rightarrow 0 = v_0 - \mu gt^* \rightarrow t^* = \frac{v_0}{\mu g}$

check dimensions of t^*: $s = \frac{m/s}{[-]m/s^2} = s$ correct
$x^* = x(t^*) = v_0\frac{v_0}{\mu g} - \frac{1}{2}g\mu\left(\frac{v_0}{\mu g}\right)^2 = \frac{v_0^2}{2\mu g}$ dimensions: $m = \frac{m^2/s^2}{[-]m/s^2} = m$ correct

$$W_{12} = \int_0^{x^*} F_f \, dx = -\mu mg \int_0^{x^*} dx = -\mu mg \, \frac{v_0^2}{2\mu g} = -\frac{1}{2}mv_0^2$$

0^2

d) generally: $W_{12} = E_{kin,2} - E_{kin,1}$

in this case: $W_{12} = E_{kin,2} - E_{kin,1} = 0 - \frac{1}{2}mv_0^2 = -\frac{1}{2}mv_0^2$
which is indeed the same as we calculated in (c).