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# Conformal Counterfactual Explanations

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## Abstract

We propose Conformal Counterfactual Explanations: an effortless and rigorous way to produce plausible and conformal Counterfactual Explanations for Black Box Models using Conformal Prediction. To address the need for plausible explanations, existing work has primarily relied on surrogate models to learn the data-generating process. This effectively reallocates the task of learning realistic representations of the data from the model itself to the surrogate. Consequently, the generated explanations may look plausible to humans but not necessarily conform with the behaviour of the Black Box Model. We formalise this notion through the introduction of new evaluation measures. In order to still address the need for plausibility, we build on a recent approach that works by minimizing predictive model uncertainty. Using differentiable Conformal Prediction, we relax the previous assumption that the Black Box Model can produce predictive uncertainty estimates.

## 1 Introduction

Counterfactual Explanations are a powerful, flexible and intuitive way to not only explain Black Box Models but also enable affected individuals to challenge them through the means of Algorithmic Recourse. Instead of opening the black box, Counterfactual Explanations work under the premise of strategically perturbing model inputs to understand model behaviour [24]. Intuitively speaking, we generate explanations in this context by asking simple what-if questions of the following nature: ‘Our credit risk model currently predicts that this individual’s credit profile is too risky to offer them a loan. What if they reduced their monthly expenditures by 10%? Will our model then predict that the individual is credit-worthy?’

This is typically implemented by defining a target outcome  $t \in \mathcal{Y}$  for some individual  $x \in \mathcal{X}$ , for which the model  $M_\theta : \mathcal{X} \mapsto \mathcal{Y}$  initially predicts a different outcome:  $M_\theta(x) \neq t$ . Counterfactuals are then searched by minimizing a loss function that compares the predicted model output to the target outcome:  $y_{\text{loss}}(M_\theta(x), t)$ . Since Counterfactual Explanations (CE) work directly with the Black Box Model, they always have full local fidelity by construction. Fidelity is defined as the degree to which explanations approximate the predictions of the Black Box Model. This arguably one of the most important evaluation metrics for model explanations, since any explanation that explains a prediction not actually made by the model is useless [13].

In situations where full fidelity is a requirement, CE therefore offers a more appropriate solution to Explainable Artificial Intelligence (XAI) than other popular approaches like LIME [19] and SHAP [10], which involve local surrogate models. But even full fidelity is not a sufficient condition for ensuring that an explanation adequately describes the behaviour of a model. That is because two

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very distinct explanations can both lead to the same model prediction, especially when dealing with heavily parameterized models:

[...] deep neural networks are typically very underspecified by the available data, and [...] parameters [therefore] correspond to a diverse variety of compelling explanations for the data. — Wilson [25]

When people talk about Black Box Models, this is usually the type of model they have in mind.

In the context of CE, the idea that no two explanations are the same arises almost naturally. Even the baseline approach proposed by Wachter et al. [24] can yield a diverse set of explanations if counterfactuals are initialised randomly. This multiplicity of explanations has not only been acknowledged in the literature but positively embraced: since individuals seeking Algorithmic Recourse (AR) have unique preferences, Mothilal et al. [14], for example, have prescribed *diversity* as an explicit goal for counterfactuals. More generally, the literature on CE and AR has brought forward a myriad of desiderata for explanations, which we will discuss in more detail in the following section.

## 2 From Adversarial Examples to Plausible Explanations

Most state-of-the-art approaches to generating Counterfactual Explanations rely on gradient descent to optimize different flavours of the same counterfactual search objective,

$$\mathbf{s}' = \arg \min_{\mathbf{s}' \in \mathcal{S}} \{ \text{yloss}(M_\theta(f(\mathbf{s}')), y^*) + \lambda \text{cost}(f(\mathbf{s}')) \} \quad (1)$$

where  $\text{yloss}$  denotes the primary loss function already introduced above and  $\text{cost}$  is either a single penalty or a collection of penalties that are used to impose constraints through regularization. Following the convention in Altmeyer et al. [1] we use  $\mathbf{s}' = \{s_k\}_K$  to denote the vector  $K$ -dimensional array of counterfactual states. This is to explicitly account for the fact that we can generate multiple counterfactuals, as with DiCE [14], and may choose to traverse a latent representation  $\mathcal{Z}$  of the feature space  $\mathcal{X}$ , as we will discuss further below.

Solutions to Equation 1 are considered valid as soon as the predicted label matches the target label. A stripped-down counterfactual explanation is therefore little different from an adversarial example. In Figure 1, for example, we have the baseline approach proposed in Wachter et al. [24] to MNIST data (centre panel). This approach solves Equation 1 through gradient-descent in the feature space with a penalty for the distance between the factual  $x$  and the counterfactual  $x'$ . The underlying classifier  $M_\theta$  is a simple Multi-Layer Perceptron (MLP) with good test accuracy. For the generated counterfactual  $x'$  the model predicts the target label with high confidence (centre panel in Figure 1). The explanation is valid by definition, even though it looks a lot like an Adversarial Example [4]. Schut et al. [20] make the connection between Adversarial Examples and Counterfactual Explanations explicit and propose using a Jacobian-Based Saliency Map Attack to solve Equation 1. They demonstrate that this approach yields realistic and sparse counterfactuals for Bayesian, adversarially robust classifiers. Applying their approach to our simple MNIST classifier does not yield a realistic counterfactual but this one, too, is valid (right panel in Figure 1).

The crucial difference between Adversarial Examples (AE) and Counterfactual Explanations is one of intent. While an AE is intended to go unnoticed, a CE should have certain desirable properties. The literature has made this explicit by introducing various so-called *desiderata*. To properly serve both AI practitioners and individuals affected by AI decision-making systems, counterfactuals should be sparse, proximate [24], actionable [22], diverse [14], plausible [7, 18, 20], robust [21, 17, 1] and causal [9] among other things.

Researchers have come up with various ways to meet these desiderata, which have been extensively surveyed and evaluated in various studies [23, 8, 16, 2, 6]. Perhaps unsurprisingly, the different desiderata are often positively correlated. For example, Artelt et al. [2] find that plausibility typically also leads to improved robustness. Similarly, plausibility has also been connected to causality in the sense that plausible counterfactuals respect causal relationships [11].

Arguably, the plausibility of counterfactuals has been among the primary concerns and some have focused explicitly on this goal. Joshi et al. [7], for example, were among the first to suggest that

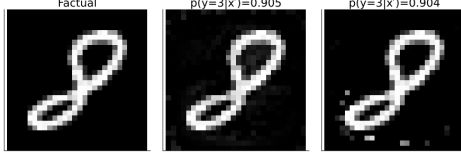


Figure 1: You may not like it, but this is what stripped-down counterfactuals look like. Counterfactuals for turning an 8 (eight) into a 3 (three): original image (left); counterfactual produced using Wachter et al. [24] (centre); and a counterfactual produced using JSMA-based approach introduced by [20].

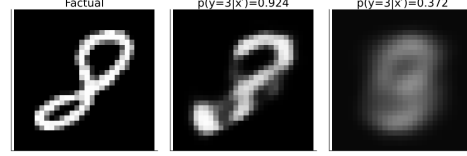


Figure 2: Using surrogates can improve plausibility, but also increases vulnerability. Counterfactuals for turning an 8 (eight) into a 3 (three): original image (left); counterfactual produced using REVISE [7] with a well-specified surrogate (centre); and a counterfactual produced using REVISE [7] with a poorly specified surrogate (right).

instead of searching counterfactuals in the feature space  $\mathcal{X}$ , we can instead traverse a latent embedding  $\mathcal{Z}$  that implicitly codifies the data generating process (DGP) of  $x \sim \mathcal{X}$ . To learn the latent embedding, they introduce a surrogate model. In particular, they propose to use the latent embedding of a Variational Autoencoder (VAE) trained to generate samples  $x^* \leftarrow \mathcal{G}(z)$  where  $\mathcal{G}$  denotes the decoder part of the VAE. Provided the surrogate model is well-trained, their proposed approach—REVISE—can yield compelling counterfactual explanations like the one in the centre panel of Figure 2.

Others have proposed similar approaches. Dombrowski et al. [3] traverse the base space of a normalizing flow to solve Equation 1, essentially relying on a different surrogate model for the generative task. Poyiadzi et al. [18] use density estimators ( $\hat{p} : \mathcal{X} \mapsto [0, 1]$ ) to constrain the counterfactual paths. Karimi et al. [9] argue that counterfactuals should comply with the causal model that generates the data. All of these different approaches share a common goal: ensuring that the generated counterfactuals comply with the true and unobserved DGP. To summarize this broad objective, we propose the following definition:

**Definition 2.1** (Plausible Counterfactuals). *Let  $\mathcal{X}|y = t$  denote the true conditional distribution of samples in the target class  $t$ . Then for  $x'$  to be considered a plausible counterfactual, we need:  $x' \sim \mathcal{X}|y = t$ .*

Note that Definition 2.1 is consistent with the notion of plausible counterfactual paths, since we can simply apply it to each counterfactual state along the path.

Surrogate models offer an obvious solution to achieve this objective. Unfortunately, surrogates also introduce a dependency: the generated explanations no longer depend exclusively on the Black Box Model itself, but also on the surrogate model. This is not necessarily problematic if the primary objective is not to explain the behaviour of the model but to offer recourse to individuals affected by it. It may become problematic even in this context if the dependency turns into a vulnerability. To illustrate this point, we have used REVISE [7] with an underfitted VAE to generate the counterfactual in the right panel of Figure 2: in this case, the decoder step of the VAE fails to yield plausible values ( $\{x' \leftarrow \mathcal{G}(z)\} \not\sim \mathcal{X}|y = t$ ) and hence the counterfactual search in the learned latent space is doomed.

### 3 A Framework for Conformal Counterfactual Explanations

In Section 2 we explained that Counterfactual Explanations work directly with Black Box Model, so fidelity is not a concern. This may explain why research has primarily focused on other desiderata, most notably plausibility (Definition 2.1). Enquiring about the plausibility of a counterfactual essentially boils down to the following question: ‘Is this counterfactual consistent with the underlying data?’ To introduce this section, we posit a related, slightly more nuanced question: ‘Is this counterfactual consistent with what the model has learned about the underlying data?’ We will argue that fidelity is not a sufficient evaluation measure to answer this question and propose a novel way to assess if explanations conform with model behaviour. Finally, we will introduce a framework for Conformal Counterfactual Explanations, that reconciles the notions of plausibility and model conformity.

### 3.1 From Fidelity to Model Conformity

The word *fidelity* stems from the Latin word ‘fidelis’, which means ‘faithful, loyal, trustworthy’ [12]. As we explained in Section 2, model explanations are considered faithful if their corresponding predictions coincide with the predictions made by the model itself. Since this definition of faithfulness is not useful in the context of Counterfactual Explanations, we propose an adapted version:

**Definition 3.1** (Conformal Counterfactuals). *Let  $\mathcal{X}_\theta|t = p_\theta(x|y = t)$  denote the conditional distribution of  $x$  in the target class  $t$ , where  $\theta$  denotes the parameters of model  $M_\theta$ . Then for  $x'$  to be considered a conformal counterfactual, we need:  $x' \sim \mathcal{X}_\theta|t$ .*

In words, conformal counterfactuals conform with what the predictive model has learned about the input data  $x$ . Since this definition works with distributional properties, it explicitly accounts for the multiplicity of explanations we discussed earlier. Except for the posterior conditional distribution  $p_\theta(x|y = t)$ , we already have access to all the ingredients in Definition 3.1.

How can we quantify  $p_\theta(x|y = t)$ ? After all, the predictive model  $M_\theta$  was trained to discriminate outputs conditional on inputs, which is a different conditional distribution:  $p_\theta(y|x)$ . Learning the distribution over inputs  $p_\theta(x|y = t)$  is a generative task that  $M_\theta$  was not explicitly trained for. In the context of Counterfactual Explanations, it is the task that existing approaches have reallocated from the model itself to a surrogate.

Fortunately, recent work by Grathwohl et al. [5] on Energy Based Models (EBM) has pointed out that there is a ‘generative model hidden within every standard discriminative model’. The authors show that we can draw samples from the posterior conditional distribution  $p_\theta(x|y)$  using Stochastic Gradient Langevin Dynamics (SGLD). In doing so, it is possible to train classifiers jointly for the discriminative task using standard cross-entropy and the generative task using SGLD. They demonstrate empirically that among other things this improves predictive uncertainty quantification for discriminative models.

To see how their proposed conditional sampling strategy can be applied in our context, note that if we fix  $y$  to our target value  $t$ , we can sample from  $p_\theta(x|y = t)$  using SGLD as follows,

$$\mathbf{x}_{j+1} \leftarrow \mathbf{x}_j - \frac{\epsilon^2}{2} \mathcal{E}(\mathbf{x}_j|y = t) + \epsilon \mathbf{r}_j, \quad j = 1, \dots, J \quad (2)$$

where  $\mathbf{r}_j \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$  is the stochastic term and the step-size  $\epsilon$  is typically polynomially decayed. The term  $\mathcal{E}(\mathbf{x}_j|y = t)$  denotes the energy function. Following Grathwohl et al. [5] we use  $\mathcal{E}(\mathbf{x}_j|y = t) = -M_\theta(x)[t]$ , that is the negative logit corresponding to the target class label  $t$ .

While  $\mathbf{x}_K$  is only guaranteed to distribute as  $p_\theta(x|y = t)$  if  $\epsilon \rightarrow 0$  and  $J \rightarrow \infty$ , the bias introduced for a small finite  $\epsilon$  is negligible in practice [15, 5]. While Grathwohl et al. [5] use Equation 2 during training, we are interested in applying the conditional sampling procedure in a post hoc fashion to any standard discriminative model. Generating multiple samples in this manner yields an empirical distribution  $\hat{\mathcal{X}}_\theta|t$ , which we can use to assess if a given counterfactual  $x'$  conforms with the model  $M_\theta$  (Definition 3.1).

**TBD**

- What exact sampler do we use? ImproperSGLD as in Grathwohl et al. [5] seems to work best.
- How exactly do we plan to quantify plausibility and conformity? Elaborate on measures.

### 3.2 Evaluation Measures

Above we have defined plausibility (2.1) and conformity (3.1) for Counterfactual Explanations. In this subsection, we introduce evaluation measures that facilitate a quantitative evaluation of counterfactuals for these objectives.

Firstly, in order to assess the plausibility of counterfactuals we adapt the implausibility metric proposed in Guidotti [6]. The authors propose to evaluate plausibility in terms of the distance of the counterfactual  $x'$  from its nearest neighbour in the target class  $t$ : the smaller this distance, the more plausible the counterfactual. Instead of focusing only on the nearest neighbour of  $x'$ , we suggest

computing the average over distances from multiple (possibly all) observed instances in the target class. Formally, for a single counterfactual, we have:

$$\text{impl} = \frac{1}{|x \in \mathcal{X}|_t} \sum_{x \in \mathcal{X}|_t} \text{dist}(x', x) \quad (3)$$

This measure is straightforward to compute and should be less sensitive to outliers in the target class than the one based on the nearest neighbour. It also gives rise to a very similar evaluation measure for conformity. We merely swap out the subsample of individuals in the target class for the empirical distribution of generated conditional samples:

$$\text{conf} = \frac{1}{|x \in \mathcal{X}_\theta|_t} \sum_{x \in \mathcal{X}_\theta|_t} \text{dist}(x', x) \quad (4)$$

As noted by Guidotti [6], these distance-based measures are simplistic and more complex alternative measures may ultimately be more appropriate for the task. For example, we considered using statistical divergence measures instead. This would involve generating not one but many counterfactuals and comparing the generated empirical distribution to the target distributions in Definitions 2.1 and 3.1. While this approach is potentially more rigorous, generating enough counterfactuals is not always practical.

### 3.3 Conformal Training meets Counterfactual Explanations

Now that we have a way of evaluating Counterfactual Explanations in terms of their plausibility and conformity, we are interested in finding a way to generate counterfactuals that are as plausible and conformal as possible. We hypothesize that a narrow focus on plausibility may come at the cost of reduced conformity. Using a surrogate model for the generative task, for example, may improve plausibility but inadvertently yield counterfactuals that are more consistent with the surrogate than the Black Box Model itself.

One way to ensure model conformity is to rely strictly on the model itself. Schut et al. [20] demonstrate that this restriction need not impede plausibility, since we can rely on predictive uncertainty estimates to guide our counterfactual search. By avoiding counterfactual paths that are associated with high predictive uncertainty, we end up generating counterfactuals for which the model  $M_\theta$  predicts the target label  $t$  with high confidence. Provided the model is well-calibrated, these counterfactuals are plausible. The authors demonstrate this empirically by evaluating

Interestingly, Schut et al. [20] point to this connection between the generative task and predictive uncertainty quantification

## 4 Experiments

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The margins in 2022 are the same as those in 2007, which allow for  $\sim 15\%$  more words in the paper compared to earlier years.

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First-level headings should be in 12-point type.

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Second-level headings should be in 10-point type.

#### 7.1.1 Headings: third level

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```
\citet{hasselmo} investigated\dots
```

produces

Hasselmo, et al. (1995) investigated...

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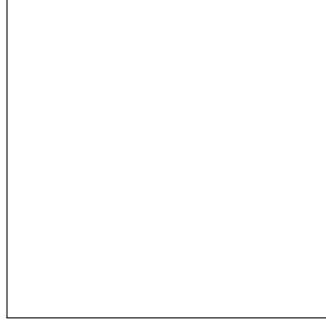


Figure 3: Sample figure caption.

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This package was used to typeset Table 1.

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<sup>2</sup>Sample of the first footnote.

<sup>3</sup>As in this example.

Table 1: Sample table title

Part		
Name	Description	Size ( $\mu\text{m}$ )
Dendrite	Input terminal	$\sim 100$
Axon	Output terminal	$\sim 10$
Soma	Cell body	up to $10^6$

## 9 Final instructions

Do not change any aspects of the formatting parameters in the style files. In particular, do not modify the width or length of the rectangle the text should fit into, and do not change font sizes (except perhaps in the **References** section; see below). Please note that pages should be numbered.

## 10 Preparing PDF files

Please prepare submission files with paper size “US Letter,” and not, for example, “A4.”

Fonts were the main cause of problems in the past years. Your PDF file must only contain Type 1 or Embedded TrueType fonts. Here are a few instructions to achieve this.

- You should directly generate PDF files using `pdflatex`.
- You can check which fonts a PDF files uses. In Acrobat Reader, select the menu Files>Document Properties>Fonts and select Show All Fonts. You can also use the program `pdffonts` which comes with `xpdf` and is available out-of-the-box on most Linux machines.
- The IEEE has recommendations for generating PDF files whose fonts are also acceptable for NeurIPS. Please see <http://www.emfield.org/icuwb2010/downloads/IEEE-PDF-SpecV32.pdf>
- `xfig` "patterned" shapes are implemented with bitmap fonts. Use "solid" shapes instead.
- The `\bbold` package almost always uses bitmap fonts. You should use the equivalent AMS Fonts:

```
\usepackage{amsfonts}
```

followed by, e.g., `\mathbb{R}`, `\mathbb{N}`, or `\mathbb{C}` for  $\mathbb{R}$ ,  $\mathbb{N}$  or  $\mathbb{C}$ . You can also use the following workaround for reals, natural and complex:

```
\newcommand{\RR}{\mathbb{R}} %real numbers
\newcommand{\Nat}{\mathbb{N}} %natural numbers
\newcommand{\CC}{\mathbb{C}} %complex numbers
```

Note that `amsfonts` is automatically loaded by the `amssymb` package.

If your file contains type 3 fonts or non embedded TrueType fonts, we will ask you to fix it.

### 10.1 Margins in L<sup>A</sup>T<sub>E</sub>X

Most of the margin problems come from figures positioned by hand using `\special` or other commands. We suggest using the command `\includegraphics` from the `graphicx` package. Always specify the figure width as a multiple of the line width as in the example below:

```
\usepackage[pdftex]{graphicx} ...
\includegraphics[width=0.8\linewidth]{myfile.pdf}
```

See Section 4.4 in the graphics bundle documentation (<http://mirrors.ctan.org/macros/latex/required/graphics/grfguide.pdf>)

A number of width problems arise when L<sup>A</sup>T<sub>E</sub>X cannot properly hyphenate a line. Please give LaTeX hyphenation hints using the `\-` command when necessary.

## Acknowledgments and Disclosure of Funding

Use unnumbered first level headings for the acknowledgments. All acknowledgments go at the end of the paper before the list of references. Moreover, you are required to declare funding (financial activities supporting the submitted work) and competing interests (related financial activities outside the submitted work). More information about this disclosure can be found at: <https://neurips.cc/Conferences/2022/PaperInformation/FundingDisclosure>.

Do **not** include this section in the anonymized submission, only in the final paper. You can use the ack environment provided in the style file to automatically hide this section in the anonymized submission.

## References

References follow the acknowledgments. Use unnumbered first-level heading for the references. Any choice of citation style is acceptable as long as you are consistent. It is permissible to reduce the font size to `small` (9 point) when listing the references. Note that the Reference section does not count towards the page limit.

## Checklist

The checklist follows the references. Please read the checklist guidelines carefully for information on how to answer these questions. For each question, change the default **[TODO]** to **[Yes]**, **[No]**, or **[N/A]**. You are strongly encouraged to include a **justification to your answer**, either by referencing the appropriate section of your paper or providing a brief inline description. For example:

- Did you include the license to the code and datasets? **[Yes]** See Section 6.
- Did you include the license to the code and datasets? **[No]** The code and the data are proprietary.
- Did you include the license to the code and datasets? **[N/A]**

Please do not modify the questions and only use the provided macros for your answers. Note that the Checklist section does not count towards the page limit. In your paper, please delete this instructions block and only keep the Checklist section heading above along with the questions/answers below.

1. For all authors...
  - (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? **[TODO]**
  - (b) Did you describe the limitations of your work? **[TODO]**
  - (c) Did you discuss any potential negative societal impacts of your work? **[TODO]**
  - (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? **[TODO]**
2. If you are including theoretical results...
  - (a) Did you state the full set of assumptions of all theoretical results? **[TODO]**
  - (b) Did you include complete proofs of all theoretical results? **[TODO]**
3. If you ran experiments...
  - (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? **[TODO]**
  - (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? **[TODO]**
  - (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? **[TODO]**
  - (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? **[TODO]**
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...

- (a) If your work uses existing assets, did you cite the creators? **[TODO]**
  - (b) Did you mention the license of the assets? **[TODO]**
  - (c) Did you include any new assets either in the supplemental material or as a URL? **[TODO]**
  - (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? **[TODO]**
  - (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? **[TODO]**
5. If you used crowdsourcing or conducted research with human subjects...
- (a) Did you include the full text of instructions given to participants and screenshots, if applicable? **[TODO]**
  - (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? **[TODO]**
  - (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? **[TODO]**

## **A Appendix**

Optionally include extra information (complete proofs, additional experiments and plots) in the appendix. This section will often be part of the supplemental material.